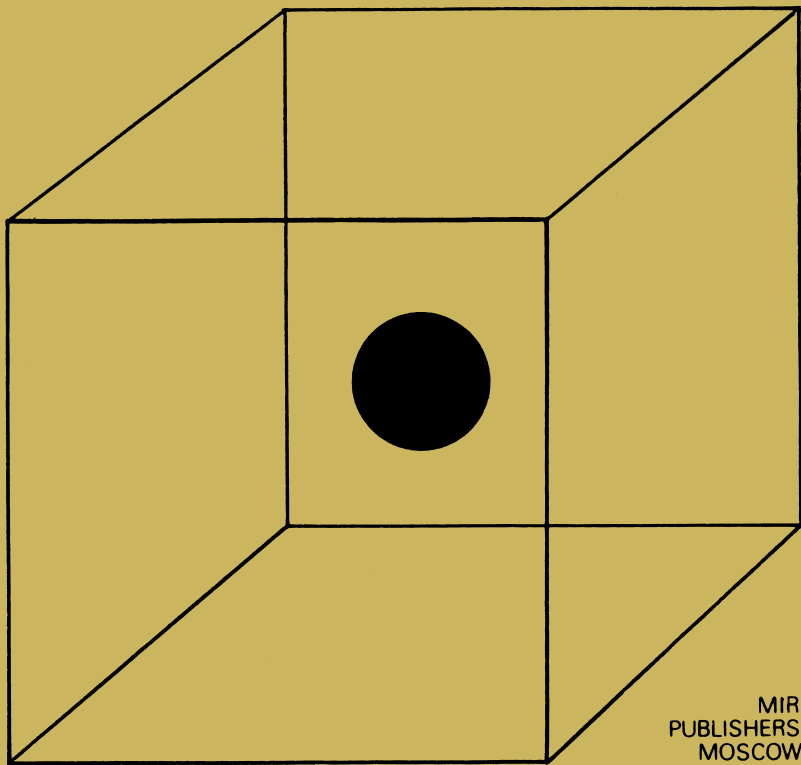


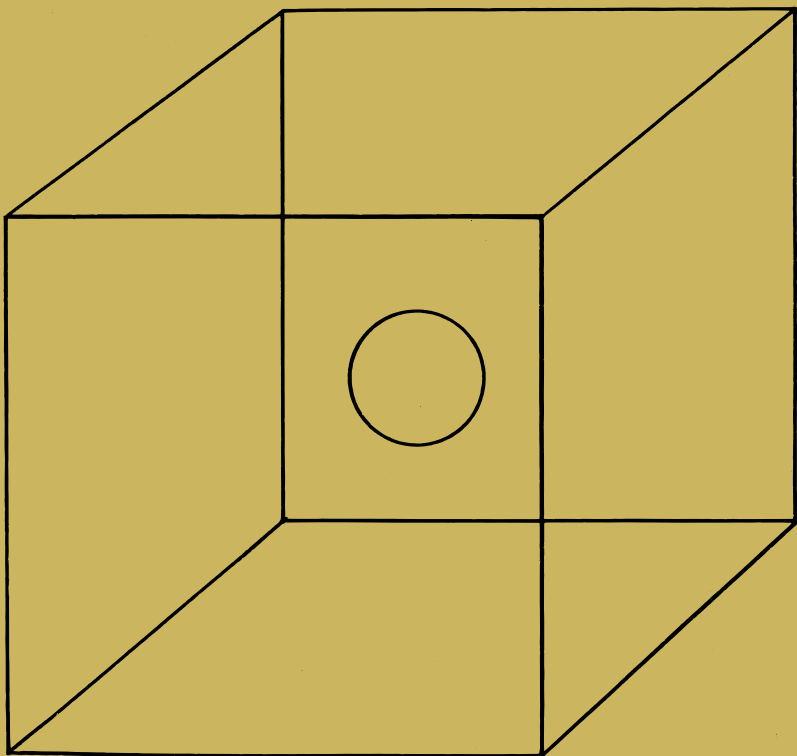
*M. I. Kaganov and I. M. Lifshits*

# *QUASI- PARTICLES*



MIR  
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MOSCOW

**This book, “Quasiparticles (Ideas and principles of solid state quantum physics)”, was written by two outstanding specialists in the solid state quantum theory. The book is meant for a broad circle of readers. It contains no complicated mathematical formulas, nor derivations of them. The authors base the presentation on analogies, assuming their readers to possess certain degree of intuition in the field of physics. The book tells how atomic particles move inside solids, what is meant by the term “thermal motion”, and how the characteristics of motion of atomic particles are revealed in the macroscopic properties of solids.**







М. И. КАГАНОВ, И. М. ЛИФШИЦ

# **КВАЗИЧАСТИЦЫ**

ИДЕИ И ПРИНЦИПЫ КВАНТОВОЙ ФИЗИКИ  
ТВЕРДОГО ТЕЛА

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Издательство «Наука» Москва

*M. I. Kaganov and I. M. Lifshits*

# *QUASIPARTICLES*

*Ideas and principles of solid  
state quantum physics*

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by V. KISSIN*

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## FOREWORD

*A number of approaches can be chosen to describe the state-of-the-arts in a sufficiently mature field of science. The choice lies with the author, and reflects his tastes, habits, and experience. This book is our attempt to present the basic concepts (or those we regard as basic) of the quantum theory of solid state, paying maximum attention to answering the question that we selected as a heading for the first chapter, viz. "What Are the Components of ..?". A favourite image the authors had invariably kept in mind was: an inquisitive boy is dismantling a toy car trying to understand what its parts are; he holds the car's skeleton in his hand, and bolts and wheels are in a pile on the floor. It was not, however, our intent to reassemble the "car" after its structure has been analyzed, and to give a detailed account of its functioning. The reader, we assume, had met with the "functioning" of solid state devices more than once; the properties of such devices make the subject of many an excellent book.*



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## *What are the components of ..?*

There exist certain concepts in science that are not easy to formulate. On their face value they are frequently accepted as obvious, although they are very profound. Careful analysis shows that these concepts embody the experience accumulated during many centuries of careful observation of nature. The statement that all material objects surrounding us possess certain structure, that we may ask "What are the components of ...?" about anything that we perceive by our senses or by means of the most sophisticated instruments, is one of such concepts. Moreover, we can expect a non-trivial, substantive answer: the object consists of molecules of atoms, ions or nucleons, photons or neutrons. In the present-day paradigm the subdivision of matter into molecules, atoms, nuclei, electrons, protons and neutrons is regarded as a truism, i. e. a self-evident truth, requiring no special proof, despite the fact that less than a century separates us from the experimental discovery of atoms and molecules. Still, we shall risk forcing an open door and take up again the question "What are the components of ...?", attempting to specify its meaning. It is known beyond doubt that all materials consist of molecules, molecules of atoms, atoms of electrons and nuclei, and nuclei of protons and neutrons; why then are we not surprised to hear that a molecule of common salt consists of sodium and chlorine *ions*, a diamond crystal consists of carbon *atoms*, and a protein macromolecule of *amino*

*acids*? Our intuition tells us that he who asks "What are the components of ...?" means decomposition into components by applying minimum force. In order to decompose common salt into ions, it is sufficient to dissolve it in water which, owing to its high polarizability, decreases coulomb attractive forces between ions so that the crystal, being a giant molecule, breaks into its components ( $\text{NaCl} \rightleftharpoons \text{Na}^+ + \text{Cl}^-$ ). Much greater forces are required to tear the electrons away from sodium or chlorine ions, roughly several electron volts (ev) per each electron. Energy a million times greater (Mev) would be needed to separate a nucleon (a proton or a neutron) from a nucleus.

Therefore, a structural unit isn't something quite unambiguous, quite definite. It essentially depends on the depth of penetration into the structure. No doubt, the discovery of quarks, those hypothetical particles making up the nucleons, would be one of the fundamental discoveries of the century; it shouldn't, however, strike the imagination of the contemporaries quite so much since it is preconditioned by the whole history of prior development of physics, which with each new discovery pushes deeper into the heart of matter its elementary essence or fundamental basis, those famous "building blocks" of which everything is built.

The meaning of such concepts as elementary particle, structural unit, and so forth, is undoubtedly conditional, and characterizes either the level to which physics has progressed or the set of means (forces) applied to single out the structural unit. However, these concepts also reflect certain objective content, permitting clear quantitative evaluation. A *molecule* can be regarded as an elementary particle in those phenomena in which it acts as a *whole*, i. e. undergoes no impacts capable of decomposing it. The bonding energy of atoms in the molecule being known, we can strictly delineate

the range of phenomena together with the values of parameters that describe them, in which the molecule is an elementary particle. Furthermore, it is always possible to specify the accuracy of the concept of an "elementary particle". For instance, at any temperature a molecular gas contains "fragments" of molecules, i. e. atoms and ions. The higher the temperature, the higher their concentration, and vice versa. The concentration of these "fragments" is a measure of accuracy of the concept of "elementary" particle with respect to a molecule in the gas.

This example demonstrates that although the choice of components is not unambiguous, it is unambiguously dictated by the specific physical conditions. One important remark: even in identical conditions one and the same object has to be considered consisting of one sort of elementary particles in experiments of one type, and of a different sort of particles in a different type of experiment. Let us take again the simple example of a molecular gas at not too high temperatures. Investigation of specific heat of the molecular gas shows that sufficiently high accuracy is obtained if the gas is assumed to consist of molecules only. A small admixture of ions can be simply neglected. In investigations of electric conductivity of the same molecular gas, however, we have to admit that the current in this molecular gas "consists" of ions. *When electric conductivity of a molecular gas is considered, the elementary particles are indeed the ions.* In such experiments neutral molecules play the role of a background. This example shows what exactly we mean by the concepts of "elementary particle", "consists of", etc.

In the case of specific heat investigation, gas molecules can be considered elementary particles because, firstly, gas energy  $E$  is, to a high degree of accuracy, the sum of energies of individual molecules; secondly,

internal motions in a molecule can be neglected when energy of an individual molecule is calculated\*.

The current must be considered consisting of ions because only ions transport charge in a molecular gas to which electric field is applied. Hence, the total current passing through the gas is the sum of elementary currents of individual ions (the elementary current of an ion is equal to its charge times its velocity).

Therefore, the concept of "elementary particle" essentially includes the *additivity* of something consisting of the introduced elementary particles, or portions.

The requirement of additivity of what a body consists of is demonstrated especially clearly for the concept of mass, if we take up the concept of mass defect, which is a corollary of Einstein's relationship  $E = mc^2$  ( $E$  is energy,  $m$  is mass, and  $c$  is velocity of light) stating that diminishing energy of a body makes it lighter. Stability of a crystalline phase signifies that the energy of the crystal is lower than the sum of energies of the separated molecules, i. e. molecules removed to infinitely large distance from one another. As an example, let us consider a crystal of solid hydrogen. Its mass is equal to the sum of masses of molecules, to the accuracy of mass defect. The mass defect in this case is very small, about  $10^{-37}$  g per one molecule. When a hydrogen molecule is "decomposed" into two atoms ( $H_2 \rightleftharpoons H + H$ ), mass defect is already about  $5 \cdot 10^{-34}$  g, and decomposition of a hydrogen atom into a proton and an electron ( $H \rightleftharpoons p + e$ ) results in mass defect of  $3 \cdot 10^{-33}$  g. This sequence of figures ( $10^{-37}$ ,  $5 \cdot 10^{-34}$ ,  $3 \cdot 10^{-33}$ ) points

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\* The possibility of neglecting internal motions in certain conditions is a corollary of quantization of energy, namely of the fact that the energy of the first excited state of the molecule is substantially greater than  $kT$  ( $T$  is gas temperature and  $k$  is the Boltzmann constant).

to the accuracy of the statement "Solid hydrogen consists of ...". It is quite natural to consider the statement "Solid hydrogen consists of hydrogen molecules  $H_2$ " as the most exact. But we must emphasize that this statement became rigorously defined only because we have analyzed one definite property of the crystal, viz. its mass. If, however, we analyzed the same statement from the "standpoint" of energy, the statement would be devoid of meaning. The energy of a crystal,  $E_{cr}$ , cannot be expressed as a sum of energies of the separate non-interacting molecules, since the energy of intermolecular interactions is far from being small in comparison with kinetic energy of the molecules.

## *Photons*

Although quantum mechanics demonstrated that waves and particles (corpuscles) are not separated by a seemingly unbridgable gap, that unsurmountable difference that was postulated by classical physics, still it is clear that electromagnetic waves and particles are qualitatively different entities. In fact, to-day we still assume without any modification the conclusion of the classical pre-quantum physics that world consists of particles and electromagnetic fields, or waves.

Quantum mechanics developed and made more profound the concepts of "wave" and "particle" by demonstrating that particles possess wave properties and waves possess corpuscle properties; it did not nullify, however, the difference between the two forms of existence of the matter (namely, field and corpuscle forms).

The space between particles, of which the material objects consist, is thus filled with electromagnetic field; this field is time-dependent, absorbed by particles and generated by the same particles. Is then the question

“What are the components of the electromagnetic field?” justified and acceptable? And if it is, what should the answer be?

It is convenient to expand the electromagnetic field into *plane monochromatic waves*, i. e. present the intensity of electric and magnetic field in any point of space at any moment of time by a sum of plane monochromatic waves with definite frequency  $\omega$  and wavelength  $\lambda$ . Electrodynamics equations lead to a conclusion that  $\omega = 2\pi c/\lambda$ , where  $c$  is the velocity of light. The procedure of expansion is mathematically rigorous. Electromagnetic field in vacuum between the bodies can be expressed exactly as the superposition (sum) of plane monochromatic waves. Note that the energy of the field is then the sum of energies of the constituent plane waves. This statement is equally applied to the energy flux. Therefore, by considering a plane monochromatic wave as an elementary wave entity, we can state that any electromagnetic field consists of plane monochromatic waves.

Corpuscular properties of electromagnetic waves are revealed in the existence of minimum portions (quanta) of electromagnetic energy. A quantum of energy,  $\varepsilon$ , is proportional to wave frequency, and the proportionality factor is the famous Planck constant  $\hbar \approx 10^{-27}$  erg·s. The energy  $E_\omega$  of an electromagnetic wave with frequency  $\omega$  is equal to an integer number of quanta  $\hbar\omega$ , i. e.  $E_\omega = n\hbar\omega$ , where  $n = 1, 2, \dots$  is an arbitrary integer. A quantum of electromagnetic energy possesses all the attributes of a quantum particle; for instance, its momentum is equal to  $2\pi\hbar/\lambda$ . The relationship between the energy and momentum of a quantum is especially simple: energy is proportional to momentum,  $\varepsilon = cp$ . We remind the reader that for an ordinary (classical) particle energy is proportional to the square of momentum:  $\varepsilon = p^2/2m$ , where  $m$  is the particle's mass.

Hence, we developed a corpuscular picture of the electromagnetic field. It consists of special corpuscles, viz. quanta, which were given the name *photons*. To describe the state of the electromagnetic field in, say, a resonator, means to state what types of photons are present and the number of photons of each type.

Photons are not merely a convenient method of describing the electromagnetic field. A large number of experimental facts cannot be interpreted at all if corpuscular concepts are neglected. Such is the photoelectric effect (photoeffect) in which an electron absorbs a photon, and the electron's energy after absorption depends on the frequency of the incident light. Or the Compton scattering of light, i. e. inelastic scattering of photons by electrons. A scattered photon transfers part of its energy to an electron, which changes the frequency of the photon. In order to emphasize the corpuscular nature of the phenomenon, we can state this in a different manner: light scattering by electrons consists in absorption by an electron of a photon with energy  $\hbar\omega$ , and generation of a photon with energy  $\hbar\omega'$ ; obviously,  $\omega' < \omega$ . The energy difference  $\hbar(\omega' - \omega)$  is absorbed by the electron, i. e. recoil is observed. A photon and an electron collide as two balls with, naturally, very different masses. It is these and other similar facts that led to the formation of the concept of photons.

It may seem that the main difference between a photon and an ordinary particle lies in the dependence of energy on momentum. This notion is readily demonstrated to be wrong. The exact relativistic formula which relates energy of a particle to its momentum is  $E = \sqrt{m^2 c^4 + c^2 p^2}$ . We see that the dependence of energy on momentum for ordinary particles and photons is almost identical for very high momenta,



i. e. for  $p \gg mc$  (ultrarelativistic case). Moreover, there exist particles whose properties are such that we can regard them as "ordinary", although their mass, as that of photons, is zero, and the dispersion law completely coincides with that for photons. These particles are neutrinos. More: photons in a waveguide are governed by a dispersion law coinciding with that for an ordinary relativistic particle:  $\hbar\omega = \sqrt{\varepsilon_0 + c^2 p^2}$ , where  $p$  is the photon's momentum along the waveguide, and  $\varepsilon_0$  is the rest energy of the photon. It is inversely proportional to squared radius of the waveguide. We can even introduce the concept of the photon's heavy mass  $m^* = \beta \frac{\hbar^2}{R^2 c^2}$  ( $\beta$  is numerical factor).

## *Bosons and fermions*

A large number of various elementary particles are known at present. They are elementary in the sense that almost no information is available about their structure. One particle differs from a particle of a different type not only in mass, but in the whole set of properties, such as the possibility to participate in certain reactions, life time, etc.

We want to emphasize here only one of the properties of elementary particles, namely their spin, a very specific property of microparticles. In an attempt to form a classical analogy of the concept of "spin", one can imagine the particle as a submicroscopic rotating top. In accordance with their spin, all particles can be subsumed into two classes. Those particles in the first class are called *fermions*, and those in the second class are called *bosons*. A particle's spin is given by  $\hbar \sqrt{s(s+1)}$ , where  $s$  is either 0, or  $1/2$ , or 1,

or  $3/2$ , etc. If  $s = 0$  or  $1$ , or  $2$ , etc., the particle is classified as a boson, and if  $s = 1/2$ , or  $3/2$ , etc., then as a fermion. Electrons, protons, neutrons, neutrinos are fermions, while photons and  $\pi$ -mesons are bosons.

Spin is an individual property, i. e. a property of each *individual* particle, but at the same time it determines the particles' behaviour in an *ensemble* (*collective*). For instance, no two fermions can be found in the same quantum state. This statement is referred to as the Pauli exclusion principle. In essence, this principle explains the great variety of atomic structures. Were it not for the Pauli principle, all the electrons in any atom would assemble in the lowest, so-called ground, state.

Bosons, on the contrary, tend to accumulate in one state. As for generation ("birth") of a new particle in a given state, a fermion can be generated only if the state is vacant, while the probability of boson generation is the higher the greater the number of particles in this state. Fermions are, so to speak, individualists, and bosons are collectivists.

It must be underlined that the effect of spin on behaviour of a number of particles cannot be reduced to any *force* interaction between them, and therefore no classical analog of this concept can be found.

## *Quantum statistics*

Realization that atomic structure is inherent to matter gave birth to a complex of sciences united by the idea of interpreting the macroscopic properties on the basis of known laws governing the motion of individual particles, i. e. to statistical physics. The picture that appears is most clear in the case of gases where interactions between particles are comparatively

weak; hence, the properties of gases can be completely described in terms that deal exclusively with the behaviour of an individual particle. Studying the properties of a gas, one can often assume that particles do not interact at all. A gas consisting of noninteracting particles is said to be *ideal*. It should be kept in mind, though, that interaction between particles is the sole reason for equilibrium onset in the gas. The smaller this interaction, the longer it takes for the equilibrium to set in. If, however, we consider equilibrium state, interaction can simply be ignored, provided it is small. It is important to remember that in gases all macroscopic characteristics (mass, energy, pressure, etc.) are, to a good approximation, additive. In essence, additivity is a necessary condition for *synthesizing*, i. e. for *deriving the macroscopic properties on the basis of studying the properties of individual particles*; indeed, the very concept of an "individual particle" includes certain independence, even if it be approximate, of the remaining particles of the material object. In this sense, ideal gas is one of the most important objects of investigation of the statistical physics.

Let us underline once again: *individual properties of particles include those properties which determine their behaviour in the interactions with other particles*. These properties are usually divided into those responsible for force interactions and are called *charges*, and those not related to force interaction and called *spins*. For example, a proton possesses, in addition to electric charge, both barionic charge and the charge describing the so-called weak interaction, i. e. various types of  $\beta$ -decay, while a neutron possesses no electric charge but has the other two charges. In addition to the ordinary spin, both protons and neutrons are characterized by the isotopic spin, which is  $+1/2$  for protons and  $-1/2$  for neutrons. The spin, as the charge, is an

individual property of a particle, imposing the laws on the particle's behaviour in an ensemble.

Such specific quantum interaction, added to force interaction, correlates the motions of atomic particles and is felt by gas particles separated by distances of the order of, or less than,  $\lambda = 2\pi\hbar/\bar{p}$ , where  $\lambda$  is the de Broglie wavelength, and  $\bar{p} \sim \sqrt{mT}$  is the mean momentum of gas particles at temperature  $T^*$ . As temperature diminishes, this distance, of course, increases. Quantum correlation made it necessary to reconsider not only the laws governing motions of individual atomic and subatomic particles, but has substantially modified the statistical physics. The law that is cited most often in this connection is the Pauli exclusion principle (see above), which states that only one fermion can be found in one state. This exclusion principle does not apply to bosons. It is natural, therefore, that the properties of gases made of bosons and those made of fermions should be substantially different. The differences are apparent, however, only at sufficiently low temperatures of the gases:  $T \lesssim n^{2/3}\hbar^2/m$ , where  $n$  is the number of gas particles per unit volume, and  $m$  is the particle's mass. The temperature  $T_0 = n^{2/3}\hbar^2/m$  is called the degeneration temperature. Note that the degeneration temperature is an essentially quantum concept ( $T_0 \sim \hbar^2$ !). The meaning of the degeneration temperature is readily understandable: at  $T = T_0$  the product of the mean thermal momentum  $\bar{p}$  of the particle by the mean distance between the particles, which is proportional to  $n^{-1/3}$ , is approximately equal to Planck's constant  $\hbar$ . The quantum character of motion becomes well-pronounced at  $T \lesssim T_0$ . In nature the quantum (degenerated) gases

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\* Hereafter we shall often use the system of units in which  $k = 1$ , i. e. temperature is measured in energy units.

are far from being rarities: electrons in metals and neutron stars are examples of quantum gases of fermions in totally different conditions.

We will have to dwell more on quantum properties of gases of elementary particles. It should be emphasized again that both fermions and bosons behave identically at high temperatures ( $T \gg T_0$ ); independence in the motion of particles is revealed, in particular, in the fact that the net energy of the gas at equilibrium is proportional to the number of particles it contains or, in other words, the *mean* energy of a particle is independent of the total number of particles in a unit volume.

We also note that in the case of ordinary gases, consisting of atoms and molecules, the degeneration temperature is so low that quantum properties have “no chance” to develop, since the gas turns into liquid or even solidifies “earlier”, i. e. at a higher temperature.

### *Gas of fermions (Fermi-Dirac degeneration)*

Decrease of temperature down to absolute zero does not stop all motion of the particles. Momenta of *all* the particles cannot be zero, since this would contradict the Pauli exclusion principle. The combination of the two conditions, viz. minimization of energy and the Pauli principle, results in the situation when fermions fill all states with momentum below the certain quantity  $p_F$ ;  $p_F$  is referred to as the boundary or Fermi momentum (Fig. 1). For particles with spin of 1/2,  $p_F = \sqrt[3]{3\pi^2 n \hbar}$ . The mean energy of a particle of the gas at  $T=0$  is equal to  $0.3(3\pi^2 n)^{3/2} \hbar^2/m$ . Notice that the mean energy of a particle is a function of the number of particles in a unit volume. This is a corollary of *quantum correlation* in the motion of particles, i. e. of the Pauli principle. At non-zero but

sufficiently low temperatures ( $T \ll T_0$ ) the state of the gas differs only slightly from that at  $T=0$  (Fig. 1); as temperature increases, a small number of particles is transferred from states with  $p < p_F$  to states with  $p > p_F$ . Obviously, the number of such particles is a function of temperature. We see that excitation of a gas of fermions is characterized by the generation of a free state, or "hole", with momentum  $p < p_F$  and of a particle with

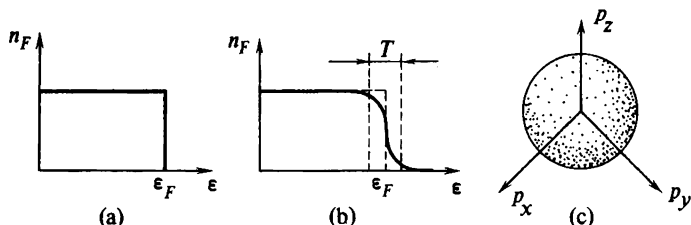


Fig. 1. (a) Fermi function at  $T=0$  is the Fermi step, with all states with  $\epsilon < \epsilon_F$  occupied ( $n_F = 1$ ) and all states with  $\epsilon > \epsilon_F$  empty ( $n_F = 0$ ). (b) Fermi function at  $T \neq 0$ ,  $T \ll \epsilon_F$ . The states in the vicinity of  $\epsilon = \epsilon_F$  are partially occupied ( $n_F \neq 0$  and  $n_F \neq 1$ ). (c) Fermi sphere (a sphere in the momentum space, of radius  $p_F = (3\pi^2 n)^{1/3} \hbar$ ) "housing" all the particles at the absolute zero

$p > p_F$ . If the number of gas particles is prescribed, then an elementary (simplest) excitation in the gas can only take the form of generation of a particle and a hole. It is said in such cases that fermions are produced in pairs, i. e. as a particle and a hole. Excitations may annihilate, i. e. the particle may occupy the free state with  $p < p_F$ , and both the particle and the hole disappear. Normally the energy of a system is measured with respect to its ground state.

Obviously, the energy of a Fermi gas at  $T \ll T_0$ , with respect to the energy of the ground state is the sum of energies of particles and holes. The mean energy of a particle (or hole) is almost independent of the number of particles (holes), i. e. there is nearly no correlation of their motions (correlation is the lower the smaller the number of particles and holes). Quantum properties of a gas of fermions are seen, in particular, in the decrease of its specific heat as temperature diminishes.

### *Gas of bosons (Bose-Einstein degeneration)*

At the absolute zero of temperature all bosons are "at rest": their momenta are zero, but the coordinates of the particles are completely indeterminate. At  $T \neq 0$  (but  $T \ll T_0$ ) almost all the particles have zero momentum and only a small number of them move, i. e. have non-zero momentum. The number of moving particles (those with  $p \neq 0$ ) is the higher the higher the temperature (their number is proportional to  $T^{3/2}$ ). All the particles move only at temperatures above  $T_B = 3.31 T_0$ ;  $T_B$  is the degeneration temperature of an ideal gas of bosons with zero particle spin.

If  $T \leq T_B$ , the mean energy of gas particles depends on their number, but this dependence vanishes if only moving particles (those with  $p \neq 0$ ) are taken into account at  $T \ll T_B$ . In this case *moving* particles play the part of elementary excitation. Their number is apparently not conserved. Note that elementary excitations in a Bose-gas are generated one at a time. Quantum properties of a gas of bosons are revealed in the dependence of its specific heat  $C$  on temperature:  $C \sim T^{3/2}$ .

Consideration of the two simple examples, namely of a system of noninteracting fermions and that of

bosons, makes it clear that quantum correlation substantially modifies the properties of systems, devoiding the properties of quantum gases of clearness inherent to classical examples. The following feature must be emphasized: both the Fermi- and the Bose-gas are characterized by the degeneration temperature. Although the degeneration temperatures in Fermi- and Bose-gases are somewhat different, both are of the order of  $T_0 = n^{2/3} \hbar^2 / m$ .

At high temperatures ( $T \gg T_0$ ) quantum correlation becomes negligible, and gases demonstrate their usual properties: *mean energy of a gas is proportional to the number of particles, i. e. each particle moves independently of the remaining particles, and the equipartition law is valid.*

Particles and holes in a Fermi-gas, and moving particles in a Bose-gas are sometimes called *quasi-particles*, although it is clear that here quasiparticles are but the actual particles of the gas only enumerated in a different (more convenient) manner\*. Obviously, the number of quasiparticles is not conserved, being a function of temperature. At low temperatures ( $T \ll T_0$ ) the number of quasiparticles is small, and these quasiparticles, with negligible correlation between them, show the ordinary gas properties. At temperatures close to the degeneration temperature the properties of gases cannot be analyzed by considering the classical motion of either individual particles or quasiparticles. Although no force interaction is present between the particles, their motions are correlated and a clear classical gas picture is incorrect; nor is reasoning in terms of quasiparticles saving the situation, since

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\* We will introduce later the generally accepted definition of a "quasiparticle" and formulate the difference between particles and quasiparticles.



quasiparticles are too numerous and the motion of one quasiparticle depends on motions of all the remaining ones. We shall ascertain later that this situation is essentially typical for all macroscopic systems: a temperature range can always be found in which the internal energy of a solid cannot be expressed as a sum of energies of particles or quasiparticles with uncorrelated motions.

## *Energy spectrum*

Each physical system is characterized by a definite energy spectrum. In a hydrogen atom, for instance, the electron's energy may assume the values given by

$$E_n = -E_0/n^2, \quad E_0 = 13.5 \text{ eV}, \quad n = 1, 2, \dots$$

The above values of energy are approximate, since slight interactions are neglected. Moreover, we did not take into account the atom's motion as a whole; this adds to energy the term  $\vec{P}^2/2M$ , where  $\vec{P}$  is the atom's momentum, and  $M$  its mass. This last remark is completely immaterial, since it is always possible to change to a reference frame in which the body (the hydrogen atom in the case under consideration) is at rest. Hereafter it is always understood that this reference frame is chosen.

The knowledge of energy levels is insufficient for investigation of the properties of physical systems that will be of interest here. We must know the nature of the quantum state or states, and in the simplest case know the wave function corresponding to a given (permitted) value of energy. For example, we cannot calculate the absorption coefficient of light by the hydrogen atom, its dipole moment and other characteristics if wave functions of an electron in the atom are unknown.

So far nothing was said about positive values of electron energy, corresponding to its infinite motion. In fact, at  $E > 0$  the atom ceases to exist, since the electron has left the nucleus. Later on we will only be interested in such values of energy of the system at which it continues to exist as a whole.

We could, if we wanted, put down the approximate values of electron energies for each atom (provided these values were known), and yet not exhaust *all* the permitted energy values of this atom. Indeed, the nucleus of the atom (excluding the nucleus of the hydrogen atom) is a complicated system and can be in a number of states, viz. the ground state and excited states. If the excited state of the nucleus lives for sufficiently long time, it will correspond to certain electron states which should somewhat differ from the electron states of the atom with its nucleus in the ground state. Taking nuclear levels of energy into account makes the picture of the atomic spectrum greatly complicated. As a rule, though, excited states of the nucleus correspond to very high values of energy, many times more than electron levels. Therefore excitation of the electron shell leaves the nuclei in the ground state, so that excited states of nuclei can simply be neglected. Conversely, electron shell becomes immaterial if anything happens to the nucleus. The discussed example represents the simplest limiting case. We were comparing two different types of motion which differed so much in energies that practically they were independent. Intermediate situations are frequently encountered. The energy spectrum of a molecule, for example, is a combination of electron energy levels, energy of vibrational motion of nuclei, and energy of rotation of the molecule as a whole.

The concepts of energy spectrum and energy structure were introduced into physics by quantum mechanics.

In classical physics a system may possess *arbitrary* energy. Of course, the character of motion in classical physics may also change drastically in response to changes in energy value  $E$ . For instance, in the case of motion under the action of attractive coulomb force, inversely proportional to squared distance to the source of the force, a body's trajectory is elliptical if  $E < 0$ , parabolic if  $E = 0$ , and hyperbolic if  $E > 0$ .

Energy spectrum is very complicated even for very simple systems. We will be interested in the energy spectrum of macroscopic objects, and first of all of crystals. Some quite general statements concerning the structure of energy spectrum of solids can be formulated, despite the apparent difficulties for determination of specific energy spectra. One of the most important conclusions that quantum mechanics derived concerning macroscopic bodies was that their energy spectra have *band* structure: bands of permitted energy values are separated by bands of forbidden values (gaps).

Later on we shall dwell on this statement in more detail (see p. 30). In itself, the statement of band structure of the spectrum is not very helpful for understanding the properties of solids. It is necessary to know the character of motion corresponding to a specific value of energy. All the various types of motion of atomic particles in solids can be subsumed into *two classes* which are conveniently characterized, and here we somewhat hasten the matter, by means of *quasiparticles*. Quasiparticles corresponding to the first class of motions are fermions, and those corresponding to the second class are bosons. True, not all motions in condensed matter can be reduced to motion of quasiparticles. We shall treat this later.

The fact that various motions in a solid can be interpreted in a simple manner and a lucid image can be constructed, is surprising in itself, and calls for

explanation. At the first glance, any motion in a system of strongly interacting atoms must perturb "everything imaginable", and it should be impossible to say anything definite about such a perturbation. Simplification of motions in solids, i. e. approximate splitting of motions, is based on really existing *differences in masses*, viz. those of atomic nuclei and electrons, and *differences in forces*, viz. electric, magnetic, etc. While giving examples of various quasiparticles, i. e. types of elementary motions, we shall thereafter emphasize the physical nature of the comparative simplicity of the considered type of motion.

The ground state of a crystal, i. e. the state corresponding to the least possible energy at  $T = 0^\circ\text{K}$ , can be used as an example of stability; indeed, a crystal will remain in this state eternally, provided no external force is applied. The motion of atomic particles, however, does not stop even at absolute zero of temperature: electrons in atoms move around nuclei, and atoms oscillate around equilibrium positions, but this "zero", *quantum* motion is not *thermal*. The energy related to it must be considered as the reference level, i. e. zero energy of the solid. The majority of dynamic, and among them thermal, magnetic, galvanic and some other properties of solids are tied to the structure of its energy spectrum, to the properties of quasiparticles which realize elementary motions in solids.

## *Phonons*

The simplest type of motion in solids is, beyond any doubt, the vibratory motion of atoms and molecules of the solid around equilibrium positions. As a result of interparticle interaction, moving atoms "clutch" at the neighbour atoms. Therefore the motion of an individual atom is not an elementary form of atoms' motion

in solids. An analysis demonstrates that *waves of displacements* of atoms can be considered as such elementary form of motion. Let us clarify what this means. In fact, the mean amplitude of vibrations of atoms  $u$  is small compared to interatomic spacing  $a$  at any temperature at which a solid exists. Even at melting point the  $u/a$  ratio *does not exceed* a few percent (see below). The ratio  $u/a$  serves as a small parameter which enables to construct a qualitative, and to a high degree quantitatively correct, picture of motion of the atoms. If we retain only the terms of expansion quadratic in  $u/a$ , the energy of a solid is the sum of energies of individual waves, and the properties of waves can be easily derived if interaction forces between atoms are known. Waves of displacements, as all other waves, are characterized by wavelength  $\lambda$  and frequency  $\omega$ . Frequency is a function of wavelength and of its direction of propagation.

The periodic arrangement of atoms in a crystalline lattice results in a *periodic dependence of frequency  $\omega$  on wave vector  $\vec{k}$*  ( $\vec{k}$  is a vector with magnitude  $2\pi/\lambda$  and directed along the propagation direction).

The quantum, i. e. corpuscular, properties of waves are revealed in the existence of the minimum portion of energy of the crystal's vibration with a given frequency. This makes it possible to put in correspondence to waves the specific quasiparticles, phonons. Phonon's energy  $\epsilon$  is  $\hbar\omega$ , and its momentum  $\vec{p}$  is equal to  $\hbar\vec{k}$ . Energy is a periodic function of momentum  $\vec{p}$ .

Rigorous consideration shows that several different types of waves of atoms' displacements can propagate in crystals. They differ in the motions of atoms within one cell of a crystal. Therefore there are several types of phonons in crystals. Each type of phonons is characterized by a specific dependence of energy on momentum.

Strictly speaking,  $\bar{p}$  is not momentum. This vector, whose properties are very similar to those of momentum, is called *quasimomentum*. The most significant difference between momentum and quasimomentum is that in collisions of phonons as well as in collisions of any quasiparticles in crystals, quasimomentum may be transferred to crystal lattice by discrete portions, i. e. quasimomentum is not conserved.

No doubt, quasiparticles, phonons among them, can be introduced quite rigorously, i. e. it can be rigorously demonstrated that energy of a crystal is the sum of energies of individual quanta, phonons in this case, each quantum possessing all the attributes of a quantum particle. Phonons describe the vibratory motion of atoms in a crystal *not completely*; they represent only the *above-zero* motion. Clearly, the number of phonons is the greater the higher the temperature. This reflects the increase in amplitude of vibrations of the atoms as the temperature rises. It may seem that a phonon differs from a real particle mainly in its ability to appear or disappear (be born or perish); real particles, however, can appear or disappear as well. For example, an electron and a positron turn into a photon. When a phonon is born or disappears, the properties that are conserved are, as with real particles, energy and . . . those which must conserve. Concretization of conservation laws is a corollary of penetration into the properties of specific particles.

There is a clear distinction between phonons and real particles. This distinction is typical of *all* quasiparticles, so it is worthwhile to consider it in more detail.

Turning to biology, we find the concept of "area", i. e. of the region inhabited by a specific species of the animal world.

The areas "inhabited" by quasiparticles and real particles are absolutely different. Real particles exist,

i. e. move, collide, turn into one another, and form more or less complicated structures (from atoms to crystals) in empty space, while quasiparticles exist, i. e. move, etc., inside macroscopic systems, which are constructions of real particles. An atom or an electron may be extracted out of a crystal into vacuum, while a phonon cannot be "freed" out of a crystal, although it may leave one crystal to enter another.

Properties of particles reflect those of their areas. For example, the properties of empty space are seen in the fact that the energy of a real particle is independent of the direction in which the particle moves. *Strictly speaking, the energy of a free particle is independent of the direction of its momentum, this independence being a corollary of the isotropy of space.* The properties of all quasiparticles, not only those of phonons, reflect the properties of their area, the crystal, i. e. of the atoms, molecules or ions arranged periodically in space. We have mentioned that *periodic arrangement of atoms in crystals results in the fact that all quantities (energy among them) are periodic functions of quasimomentum.*

This argumentation was meant to underline that periodic dependence of energy on quasimomentum is a *geometric* property. Knowing the geometric structure of the crystal we could say what the periods are in the dependence of a quasiparticle's energy on its quasimomentum knowing nothing about interaction forces between the atoms and without specifying what kind of motion is discussed.

The material nature of the area of quasiparticles is seen in the existence of a selected frame of reference. This nullifies Galileo's relativity principle as far as quasiparticles are concerned. This makes the properties of quasiparticles considerably less "routine" than those of ordinary "genuine" particles.

But let us return to phonons. We have established that a wave of displacements with definite wavelength and frequency is the simplest form of motion of atoms in crystals. We have put this wave in correspondence with a quasiparticle called phonon. A phonon is an elementary *non-decomposable* motion, although it involves the motion of *all* atoms of the crystal. One significant factor must be emphasized. We have already mentioned that a phonon possesses all the attributes of a quantum particle. We even refer to the wave of displacements as the *one-particle excitation*. It should seem rather surprising that the motion of *all* particles of a crystal is described by a *single* quasimomentum. Mathematical analysis of the equations of motion of atoms in a crystal demonstrates that the reason for this simplicity lies in the periodic arrangement of atoms. Phonons cannot be introduced in amorphous materials (glass, for example), although here too the atoms oscillate around their equilibrium positions. Actually, this last statement is not quite correct. We can introduce for an amorphous body low-momentum phonons, but not high-momentum ones. The low-momentum phonons can be introduced because the atomic structure of matter becomes immaterial for long-wavelength vibrations, i. e. for wavelengths much greater than interatomic spacings: in other words, a solid in this case behaves as the elastic uniform continuum. If, however, wavelength is approximately equal to or is slightly greater than the spacings between the atoms, their spatial arrangement becomes of principal significance. Therefore, one-particle excitations, i. e. waves of displacements, or phonons, can propagate in solids only within rigorously ordered periodical structure.

Now we can specify our statement on the band structure of the energy spectrum in solids. Excitation states in crystals can be classified as follows:



*one-particle* states: a crystal contains a single quasiparticle, for example, one phonon;

*two-particle* states: two quasiparticles in a crystal;

*three-particle* states, etc.

Energy spectrum has a band structure within each *class*, i. e. consists of bands of permitted and gaps of forbidden energy values. This is one of the most important properties of the energy structure of crystals. Of course, the crystal's energy can assume *any* value, since a large number of quasiparticles can exist in the crystal at the same time.

Introduction of phonons makes it possible to express the energy of vibrational motion of atoms in a crystal as the sum of energies of individual phonons; we have already mentioned above that this justifies our treating phonons as elementary particles, or, more precisely, quasiparticles. Furthermore, to the question "What does the vibrational energy of a crystal consist of?" we are now fully entitled to answer "Of phonons..."

A solid, or let us say a crystal, is conveniently thought of as a vessel containing a gas of phonons. We artificially separate atoms from phonons, which are nothing but waves of displacements of the former, but thereby gain a possibility of treating the dynamic properties of crystals as those of a gas of phonons. True, this is not an ordinary gas since the number of particles in this gas is not conserved (indeed, the number of phonons is the greater the higher the temperature). Introduction of the gas of phonons not only makes the whole picture more lucid, but at the same time enables to apply a number of ideas of the kinetic theory of gases to the theory of solid state.

Phonons are *bosons*, which readily follows from an analysis of the vibrational motion of atoms. As always, the quantum properties of the phonon gas are well

pronounced at low temperatures, and one direct corollary of the *quantum* statistical properties of the phonon gas is the decrease in specific heat  $C$  of crystals as temperature is lowered toward zero. In the vicinity of absolute zero,  $C \sim T^3$  and may be less than specific heat at room temperature by a factor of several hundred thousand.

Specific features of quasiparticles, phonons in this case, are seen not only in the dependence of their energy on quasimomentum, but also in the character of interaction between quasiparticles. Collisions between quasiparticles will be discussed later in the book, so only a few words will be mentioned here. Two colliding phonons may disappear altogether, and a new phonon appears with the energy equal to the sum of energies of the colliding phonons. Such collisions are also possible in a gas of atoms or other real particles and not quasiparticles; chemical reactions are such collisions. Two colliding hydrogen atoms may form a molecule, while two colliding helium atoms do not form a molecule; and phonons collide mostly as described above. Moreover, a chemical reaction forms a particle of a different sort, hydrogen atoms form a molecule, while a transformation of two phonons may form a *quite similar* phonon but with higher energy. As a result of increase of the number of phonons with increasing temperature, phonons collide more often. This means that the shorter the lifetime of each individual phonon, the greater is the role played by interaction between phonons, and the further we deviate from the properties of the ideal gas of phonons; consequently, the whole picture becomes so complicated that phonons cease to be a convenient method of describing dynamic, and in particular thermal, properties of solids. Phonons are of especially little use in interpreting thermal properties of those solids which at a certain temperature

undergo a transition into a new state, i. e. a phase transition. Such transitions are events so important and so frequently encountered that deserve treatment in a special chapter later in the book.

Let us formulate an intermediate summary. Determination of the chemical structure of solids by taking into account the bonding energy, made it possible to answer the question about the constituents of a solid, or what its building material is, while introduction of phonons enabled to find out what the energy of its excited states, or at least, of some of them, is composed of. The possibility to introduce phonons, i. e. "good" long-lived quasiparticles, is based on the fact that crystalline solids almost always are at comparatively low levels of excitation. In the case of phonons this is seen in the maximum energy of phonons (the so-called Debye energy) being much smaller than the bonding energy of atoms per one particle.

The statement "A crystal is almost always at a considerably low excitation level" requires one very important qualification: "Far from phase transition points". However, phase transitions will be discussed in a separate chapter.

## *Phonons in helium*

Helium is a unique material, and its property that seems to be the most striking is its ability to remain liquid at the absolute zero of temperature. This property is quantum in nature: the amplitude of zero vibrations of helium atoms exceeds the mean distance between atoms. If helium is compressed, interatomic distance diminishes and at the same time interatomic interaction forces are increased; this reduces vibration amplitude, and at pressure of 26 atmospheres helium

solidifies. But here we are interested in *liquid* helium in the vicinity of absolute zero. Its boiling point is  $4.2^{\circ}\text{K}$ , and at  $T = 2.19^{\circ}\text{K}$  it undergoes a most unusual transformation: it sheds its viscosity and becomes *superfluid*, i. e. flows without any friction through the finest capillaries. Superfluidity will be our subject in a later chapter, and now we will take up the properties of helium at  $T < 2.19^{\circ}\text{K}$  from the standpoint of the possible atomic motions.

Sound waves can propagate in liquid helium as in any other liquid. If the wavelength of sound is large compared to interatomic distances, this is the perfectly ordinary, "normal" sound with frequency inversely proportional to the wavelength, i. e. proportional to the wave vector. If the wavelength is close to the mean interatomic spacing, the relation of frequency to wave vector becomes more complicated (Fig. 2), and it is a known fact that the curve  $\omega = \omega(k)$  has minimum at  $k = k_0 = 1.9 \cdot 10^8 \text{ cm}^{-1}$ . The quantum character of waves in helium can be described by introduction of special quasiparticles, namely phonons whose energy and momentum can be obtained, as always, from de Broglie's relations:  $\varepsilon = \hbar\omega$ ,  $\vec{p} = \hbar\vec{k}$ . Phonons with momentum close to  $p_0 = \hbar k_0$ , were given a special name — *rotons*. A phonon is characterized by a real momentum, not by quasimomentum. Helium is a liquid. Therefore the area of quasiparticles in helium is a uniform isotropic space, although it is not empty. Motion in this space can be characterized by a definite momentum. Let us underline another important property of quasiparticles in helium: their velocity,  $v = \varepsilon/p$ , is not equal to zero for any  $p$ . Even quasiparticles with arbitrary small energies, for  $\varepsilon \rightarrow 0$ , possess finite (and quite high!) velocity, equal to the velocity of sound, i. e.  $\approx 235 \text{ m/s}$ .

Waves can propagate in any liquid, not only in

helium. Specific behaviour of helium at nearly zero temperatures (below  $2.19^{\circ}\text{K}$ ) consists in that *no other macroscopic motions of atoms are possible in it*\*. Thermal motions of atoms in helium is nothing but random motion of phonons-rotons whose number obviously

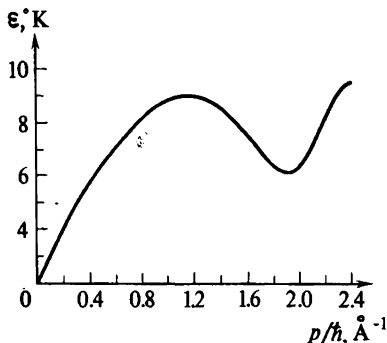


Fig. 2. The spectrum of elementary excitations (of quasiparticles) in superfluid helium. Quasiparticles with energy  $\epsilon$  being a linear function of momentum  $p$  are called phonons. The phonons' velocity is 235 m/s. Quasiparticles with momenta close to  $p_0 = 1.9 \cdot 10^8 \hbar$  are called rotons. The dispersion law of rotons is  $\epsilon \approx \Delta + (p - p_0)^2 / 2\mu_r$ , where the effective mass of a roton,  $\mu_r$ , is 0.16 of the mass of a helium atom

increases with increasing temperature. Thermal properties of helium are well known. The measured temperature dependence of specific heat and other thermodynamic characteristics of helium are in perfect agreement with the formulas which were derived on the basis of the concepts of phonons as a Bose-gas of quasiparticles.

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\* In fact, rotating helium may exhibit a specific vortex motion. It cannot be described by means of phonons. But in essence, generation of vortices in helium signifies local destruction of superfluidity.

## *Magnons, etc.*

Vibrational processes in solids are much varied. Not only atoms or molecules can vibrate around their equilibrium positions. A simple rule can be formulated: *if a classical oscillatory process is possible in a condensed matter, the quasiparticle corresponding to it is a boson.* The laws of propagation of a classical oscillatory process can be derived by means of the formulas of classical physics. Application of classical concepts is always justified if vibration energy is much greater than energy of one quantum. This comment is meant to clarify the nature of the rule. For a classical oscillatory process to exist, it is necessary that there be a multi-quantum state with a definite wave vector, i. e. a state with a large number of identical quasiparticles. This is possible only for bosons. The opposite statement is not correct: there are bosons (quasiparticles) having no classical analogs (see below).

Phonons in crystals and in helium present the simplest examples of the above-formulated rule. As the next in simplicity of genesis (origin) of quasiparticles, we can point to magnetic oscillations in ferro- and antiferromagnetics, which at the absolute zero are characterized by strictly ordered arrangement of all magnetic moments of the atoms.

Since magnetic moments of neighbour atoms interact with one another, magnetic moments of individual atoms cannot oscillate independently from those of their neighbours. An analysis demonstrates that the simplest irreducible form of motion of magnetic moments is a wave; if its wavelength is much greater than the interatomic spacing, then the frequency of the wave is proportional to the square of the wave vector in the case of ferromagnetics, and to its first power in the case of antiferromagnetics.

Waves of oscillation of magnetic moments are called *spin waves*, and the corresponding quasiparticles are called *magnons*. As follows from the above-formulated rule, magnons are bosons.

Magnons in ferromagnetics are especially similar to ordinary non-relativistic particles. Indeed, their energy  $\varepsilon$  is proportional to  $p^2$  ( $\bar{p} = \hbar k$ ). The similarity is immediately apparent if we write  $\varepsilon = p^2/2m^*$ . The quantity  $m^*$  is called the effective mass. It is the smaller, i. e. the magnon is the "lighter" (*sic!*), the stronger the interaction between neighbour atoms. Magnons make it possible to describe in terms of gas concepts not only thermal but certain specific magnetic properties of ferro and antiferromagnetics as well. For example, the magnetic moment of a ferromagnetic at a finite temperature is lower than that at the absolute zero. Indeed, an analysis shows that each magnon has a magnetic moment directed against the ordered magnetic moments of the atoms. This means that a ferromagnetic with a single spin wave propagating in it possesses the spin smaller than that at  $T = 0^\circ\text{K}$  by unity; at the same time, we have to assign to the spin wave a magnetic moment directed against the magnetic moment of all the atoms and with the value equal to unity spin. This statement becomes clear if we analyze the microscopic model of a spin wave in a ferromagnetic. We assume, for the sake of simplicity, that the ferromagnetic consists of atoms with spins equal to  $1/2$ . In this case a spin wave is propagation along the crystal of a "turned-over" spin. Naturally, the greater the number of magnons, the smaller the net magnetic moment of the ferromagnetic. And the number of magnons rises with increasing temperature. Magnons are totally absent at absolute zero.

Magnetic properties of materials cannot be reduced to those of a gas of magnons, because ordered

magnetic moments (in other words, a body without magnons) have their own magnetic properties. For instance, the whole system of atomic magnetic moments can be reoriented by placing the body in an external magnetic field.

We shall not list here other representatives of the family of bosons related to classical oscillatory motions. Although their properties are much varied, and although they serve to describe very dissimilar phenomena taking place in crystals, in essence they are very much alike. If we restrict ourselves to statements of general character, as we have to do, of necessity, in this brochure, the listing of various quasiparticles would result in repetitions.

### *Electrons. Energy bands*

Quantum mechanics has found the reason for the existence of materials with different electrical natures, i. e. it answered the question as to why some materials, viz. metals, are conductors of electric current, while other materials, viz. insulators, are not. This classification also found place for solids with intermediate properties, i. e. semiconductors and semimetals.

We have already mentioned in passing that energy spectrum of solids has a band structure. Let us have a closer look at it.

Electrons in an atom have certain discrete values of energy, normally referred to as energy levels. When atoms approach one another, it becomes possible for electrons to swap places. It must be remembered that microparticles can pass through potential barriers, i. e. the regions in space where potential energy is *higher* than the total energy of the particle. As a result of such transitions identical energy levels split, and the difference between the neighbour energy levels is



determined by the energy of interatomic interaction, in other words, by the ability of electrons to "move" from one atom into another. If there are only two electrons, as is the case for two hydrogen atoms, they occupy the lowest (ground) level; of course, their spins must be antiparallel. The best phrase to use here should be: and so on. When three atoms are brought together (assume them to be hydrogen atoms, as above), each level is split into three levels, four atoms give four levels, .... Since electrons are subject to the Pauli principle, only two of them can occupy each of the levels.

Splitting of a level into  $N$  sublevels does not require that *all*  $N$  atoms be close to one another; it is sufficient if each one can be reached via the others, from neighbour to neighbour. Maximum splitting is obviously determined by interaction of neighbour atoms. The number of atoms in a crystal is fantastically large:  $10^{22}$  to  $10^{23}$  per cubic centimeter. Each atomic level splits into  $N$  sublevels, and the intervals between them are the smaller, the greater  $N$ . In the limit, i. e. for  $N \rightarrow \infty$  the sublevels merge, or rather stick together, into a band of permitted values of energy; this band is the wider, the stronger the interaction between neighbour atoms (Fig. 3). Each level in the band is occupied by not more than two electrons, and the whole band — by  $2N$  electrons. This is a very important factor!

The state of an electron in a band is similar to both its state in an atom, and to its free state, because it can pass from one atom to another\*.

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\* Ya.I. Frenkel suggested a very good and instructive image: the motion of a point on the rim of a rolling wheel (for example, of a nail's head) resembles the motion of an electron in a solid, with rotation being the analog of the electron within the atom, and rolling being the analog of its motion from atom to atom.

Transfer of an electron from one atom to another along the sequence of lattice atoms is described by quasimomentum which has already been mentioned above. Quasimomentum, in the sense of describing an electron's motion, is very much similar to momentum.

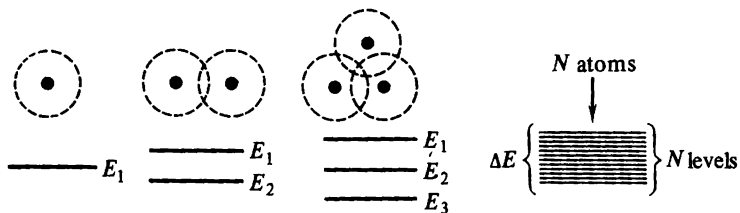


Fig. 3. Splitting of atomic energy levels due to interatomic interaction. An atomic level splits into sublevels  $E_n$ , whose number is equal to that of the atoms. The maximum value of splitting,  $\Delta E$ , is approximately equal to the energy of interaction between two neighbour atoms. Energy sublevels form a band of permitted energy levels in the crystal. Due to the Pauli principle each level can "house" two electrons. The maximum number of electrons in the band is  $2N$ , where  $N$  is the number of atoms in the crystal

In particular, an electron's velocity in a crystal is the derivative of energy over *quasimomentum* ( $v = d\varepsilon/dp$ ). Therefore, an electron's state in a crystal is given by stating to which specific band it belongs, and by giving its quasimomentum which fixes its energy within the band. We repeat, although it has already been mentioned, that energy is a periodic function of quasimomentum.

## $\bar{p}$ -Space

Because of the vector character of quasimomentum, it is convenient to operate in terms of the space of quasimomentum, or for brevity,  $\bar{p}$ -space. Each band has a corresponding  $\bar{p}$ -space. The expressions "an

electron's quasimomentum is  $\vec{p}$ " and "an electron is in the point  $\vec{p}$ " are identical in meaning.

Let us consider a crystal in the ground state, i. e. at the absolute zero temperature. All its electrons occupy available states in a manner satisfying two principles: the total energy must be minimal, and each state, i. e. each definite value of quasimomentum  $\vec{p}$  in a definite band must be occupied by not more than two electrons (this follows from the Pauli principle). Assume that each atom contains  $z$  electrons. Then the total number of electrons in the crystal is  $Nz$ . It seems logical that  $Nz$  electrons can fill either  $z/2$  bands if  $z$  is even, or fill one of these bands only in half if  $z$  is odd. Unfortunately, our "either ... or" reasoning yields an oversimplified picture: bands may overlap (Fig. 4). Therefore in most cases a crystal has totally filled bands, partially filled bands, and totally empty bands. Sometimes partially filled bands are absent. The materials with bands partially filled with electrons are metals, or conductors. Materials without such bands are insulators, or dielectrics. We are to prove this in a few lines, and now let us consider how the  $\vec{p}$ -space, and not the bands, is filled. The fillings are, of course, identical. If the band is filled with electrons, the  $\vec{p}$ -space of this band is uniformly filled: there are two electrons in each point of the  $\vec{p}$ -space; if the band is only partially filled, the  $\vec{p}$ -space contains regions free of electrons. It is clear that the boundary which separates the region of the  $\vec{p}$ -space occupied by electrons from the free region is the surface of equal energy. Were it not so, we could reduce the net energy of the crystal without violating the above-formulated principles. The boundary surface is called the *Fermi surface*, and the corresponding energy is called the *Fermi energy*. In a gas of free fermions the Fermi surface in the  $\vec{p}$ -space is a sphere.

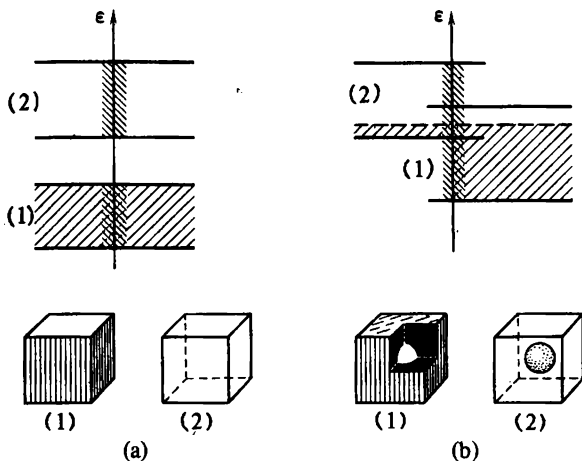


Fig. 4. (a) The permitted energy bands (marked by hatching along the axis) do not overlap. The lower hatched energy band is filled by  $2N$  electrons in the ground state, while the upper band is empty. (b) The permitted energy bands overlap, and  $2N$  electrons are both in the lower and the upper bands.

Since energy is a periodic function of quasimomentum, we can restrict ourselves to one unit cell while drawing  $\bar{p}$ -space. If the first permitted band is completely filled, its  $\bar{p}$ -space is uniformly filled;  $\bar{p}$ -space of the second band contains no electrons (case (a)). In the case (b) electrons are present both in the  $\bar{p}$ -space of the first band and in that of the second band. The first band is almost filled while the second is nearly empty

### *Metals (conductors), insulators, semiconductors, and semimetals*

The passage of current through matter is a complicated process whose description required thorough investigation of the behaviour of charged particles when electric field is applied to the material. Current may be transported both by electrons and by "heavy

charged particles", for example by ions in electrolytes. We, however, will be interested here only in *electron* conductors, i. e. in solids in which ions are arranged in a crystalline lattice while electrons can move through the lattice from ion to ion comparatively freely. When no electric field is applied to the body (in other words, the conductor is not connected to the voltage source), the number of electrons moving in one direction is exactly equal to that of electrons moving in the opposite direction, so that the current is zero. Electric field accelerates the electrons. In more precise terms, the change in quasimomentum of an electron per unit time is equal to the force applied to this electron by the electric field. A steady-state process would be impossible if electrons could not get rid of the quasimomentum "passed on" to them by the field. An analysis shows that any irregularities in the crystal, such as impurity atoms, grain boundaries, dislocations, sample surface, etc., serve as "sinks" for quasimomentum of the electrons. Even an ideal crystal contains at any temperature unavoidable irregularities, viz. quasiparticles which collide with electrons and thus let them "get rid" of the quasimomentum imparted to them by the electric field. In the process of obtaining quasimomentum from the field and transferring it to the crystal in collisions with irregularities, electrons "drift" in the field, i. e. they slowly move in the direction of the field\*. This picture allows to speak about elementary acts of electronic conduction. These are: gaining of quasimomentum between collisions due to the force applied, and its loss via collisions.

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\* *Slowly* here is not just a figure of speech. The mean velocity of electrons in copper, in which the current passes, at room temperature and with current density of  $1 \text{ a/cm}^2$ , is  $0.2 \cdot 10^{-2} \text{ mm/s}$ .

Electrons can change their quasimomentum only if empty sites are available for them in the  $\bar{p}$ -space. If, however, the  $\bar{p}$ -space is completely filled with electrons, this process becomes impossible. Hence it is clear that crystals with partially filled bands must, in the ground state, i. e. at the absolute zero temperature, conduct electric current. These are *metals*. And crystals

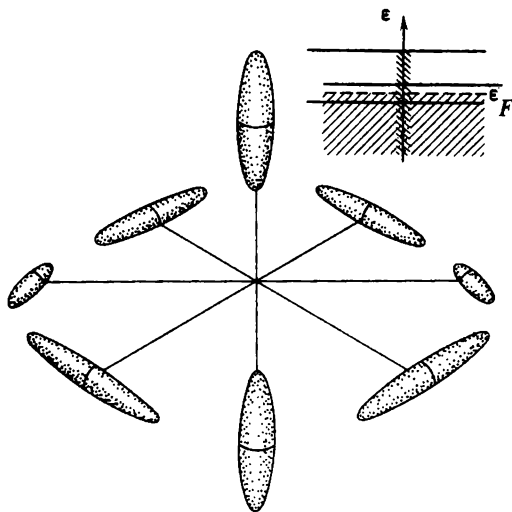


Fig. 5. Energy bands in Bi slightly overlap. Were it not for this overlapping, Bi would be an insulator. The Fermi surface consists of three ellipsoids in the upper band and one in the lower band. Their positions in  $\bar{p}$ -space are different. All states outside the ellipsoids in the upper band are unoccupied while in the lower band unoccupied states are inside the ellipsoids. All four ellipsoids are usually shown on the same drawing (two unit cells are shown in this figure). The upper-band ellipsoids are distinguished by their anisotropy (the lengths of the ellipsoid axes are related as 1:0.1:0.05). The number of occupied states in the upper band, equal to the number of unoccupied states in the lower band, is  $\approx 3 \cdot 10^{17}$  electrons per  $\text{cm}^3$ .

which at  $T=0^\circ\text{K}$  have only completely filled and completely empty bands are *insulators*.

Those insulators in which, upon heat excitation, an appreciable number of electrons can get into the empty band, are comparatively good conductors of current at temperatures above absolute zero. These are *semiconductors*.

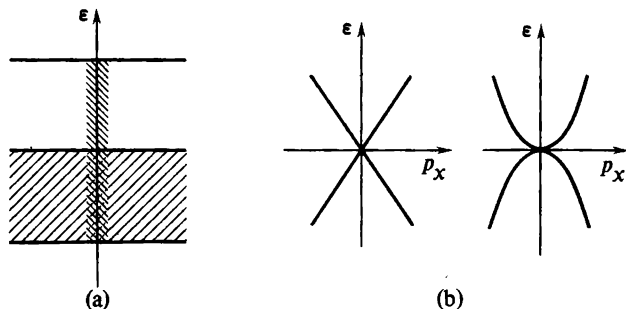


Fig. 6. (a) The lower filled band in zero-gap semiconductors borders on the upper band which has no electrons at  $T=0^\circ\text{K}$ . (b) Possible types of dependence of the energies of electrons and holes on the projection of quasimomentum on the  $x$ -axis ( $p_y = p_z = 0$ ) (see the caption to Fig. 7)

It is not infrequent that at absolute zero the bands overlap though only slightly (Fig. 5). As a result, the number of electrons and unoccupied sites in the partially filled bands is very low, of the order of  $10^{-3} - 10^{-5}$  per atom. Such materials possess properties intermediate between those of metals and semiconductors: they behave as metals at low temperatures and as semiconductors at high temperatures. These materials are often called *semimetals*. Typical examples are bismuth and antimony. Finally, it was found out recently

that there exists a whole class of materials which occupy precisely the intermediate position between metals and semiconductors. These are *zero-gap semiconductors*, i. e. crystals with zero distance between the filled and the empty bands (Fig. 6).

To summarize: metals have a band (or bands) partially filled with electrons. The boundary in the

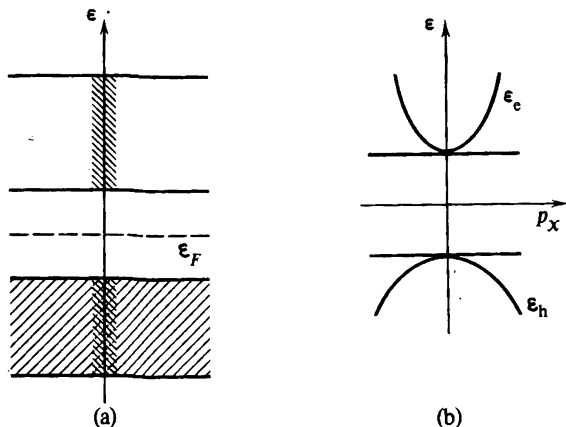


Fig. 7. (a) Energy structure of an intrinsic semiconductor with no impurities. (b) If one electron is in the conduction band, one "hole" is formed in the valence band. The drawing plots the electron energy,  $\epsilon_c$ , and the hole energy,  $\epsilon_h$ , as functions of quasimomentum projection  $p_x$  for  $p_y = p_z = 0$ . Energy of the crystal  $E_0$ , measured with reference to the ground state  $E_0$ , equals the electron energy minus the hole energy ( $E = E_0 + \epsilon_c - \epsilon_h$ )

$\bar{p}$ -space, separating filled states from free states is the Fermi surface. In semimetals the volume inside the Fermi surface is small compared to the volume of a unit cell in the  $\bar{p}$ -space. In zero-gap semiconductors the Fermi surface is a line or a point. A formal



analysis demonstrates that semiconductors can be characterized by the Fermi energy located exactly in the middle of the forbidden energy gap (Fig. 7). Obviously, no meaningful Fermi surface can be introduced in a semiconductor.

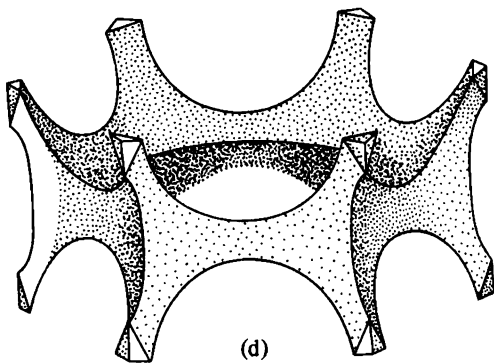
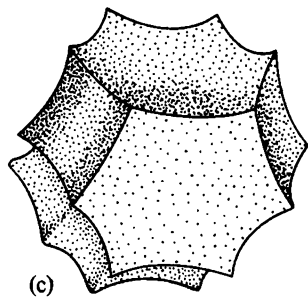
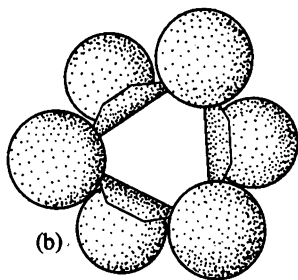
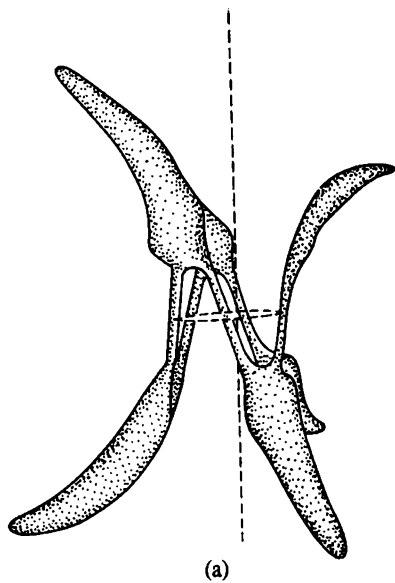
## *Electrons and holes*

We have given above a comparatively detailed description of the ground state of the electron system in a crystal. As always, excitation of the electron system is related to generation of elementary excitations, i. e. quasiparticles which realize the simplest forms of motion in solids. The electron system, like a crystal as a whole, is a source of elementary excitations of two types: fermions and bosons. By their nature fermions are close to an individual free electron which does not interact with its ilk, while excitations of boson type resemble waves (this will be discussed later in the book).

Let us neglect for a time the electron-electron interaction, and consider each electron as moving only in the force field of the ions. The complicated dependence of energy on quasimomentum, resulting in complicated shapes of the Fermi surface in metals (Fig. 8), is a corollary of the periodic forces acting on each electron from the ions. Atoms' nuclei are thousands of times heavier than electrons and therefore move much more slowly. Hence, nuclei (or ions) can be considered immovable when the electron subsystem of a solid is considered.

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*Fig. 8. Examples of portions of Fermi surfaces of a number of metals: (a) Sb; (b) W; (c) Al, In; (d) Zn. Fermi surfaces of all metals are known nowadays, and for most of the metals all details of their Fermi surfaces were measured with sufficient accuracy*



In this simplest case an elementary excitation of the electron system means that an electron gains energy, thus moving over to some point of the  $\bar{p}$ -space which was unoccupied prior to excitation. If a metal is involved, the electron may remain in the same band and increase its energy, without passing into another band of the  $\bar{p}$ -space. Note that this increment may be infinitesimal for electrons with energy equal to the Fermi energy. In semiconductors, even zero-gap semiconductors, an electron increasing its energy necessarily leaves its band for another. As a rule, this required a finite energy gain. Obvious exception from this rule is constituted by zero-gap semiconductors.

And so, one electron is transferred to a point in the  $\bar{p}$ -space which was unoccupied. Therefore, an unoccupied site, or hole, appears in the formerly occupied point of the  $\bar{p}$ -space. The absence of charge in a point where charge is to be located in the case of uniform filling is a perfectly observable event. A hole behaves as a positively charged particle. This statement has no mysticism about it.

When field forces all electrons to drift to the *right*, a hole is in fact moving to the *left with respect to the background*. The background, i. e. the electron system in its ground state, can be completely neglected in many cases. Therefore only those electrons that left the background, i. e. gained certain energy, are labelled as electrons or particles.

One of the basic properties of excitation of Fermi type is to generate pairs of particles: an electron and a hole. A hole plays the role of an antiparticle with respect to an electron. The two may even annihilate (the meaning is clear to the reader: an electron returns to its "proper place" and eliminates both the particle and the hole).

The states of crystals which often are of maximum interest are those close to the ground state, so that maximum attention in metals is paid to the Fermi surface and its vicinity, and in semiconductors\* to the upper boundary of the last filled band (referred to as valency band) and to the lower boundary of the first empty band (conduction band).

### *Landau Fermi liquid*

Obviously, electrons may be treated as noninteracting only for the sake of convenience. It might seem that this approach is totally unfounded. For instance, a singly ionized ion, e. g. sodium ion, produces around it an electric field not a bit stronger than that of an electron turned loose. Why then should we take into account the electric field of the ion lattice and try to neglect the interaction between electrons?

One possible explanation, simplified to the extreme, is like this. When describing a field of forces acting on an electron, we do not give any details, such as the magnitude of forces, dependence on coordinates, etc. Let us picture this field as a result of combined action of the ions and *all electrons without one*. Namely, without the one whose motion we discuss. Since a solid contains a tremendous number of electrons, removal of one electron does not change the properties of a solid, among them its symmetry. And it is essential that a number of conclusions, at any rate all those we mention in this book, are obtained by taking into account only the symmetry properties of crystals.

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\* If electron properties are discussed, even the best insulators are treated as semiconductors. At any rate, no generally accepted rule was suggested to distinguish insulators from semiconductors.

Motion of electrons in the midst of other electrons has a specific feature which must be mentioned even in as brief and superfluous manner of presentation as the one chosen here. Namely, interaction of the particles results in the dependence of motion of an *individual* electron upon the states of *all* electrons; the first apparent reason is that no two particles can be found in one state (this is forbidden by Pauli's principle). L. D. Landau developed a special mathematical method for describing phenomena caused by interaction between Fermi particles. This chapter of theoretical physics was named the theory of Fermi liquid. First it was developed to explain properties of the lighter isotope of helium ( $\text{He}^3$ ) in the temperature range of  $0.01\text{--}0.1^\circ\text{K}$ . At lower temperatures, i. e. at millidegrees,  $\text{He}^3$  undergoes transition into superfluid state, and cannot be described by the Landau theory of Fermi liquid. Later the theory of Fermi liquid was adapted to describe conduction electrons in metals. In semimetals the Fermi-liquid effects are negligibly small.

A characteristic feature of the theory of Fermi liquid, at least that of interest to us, is that interaction of fermions with each other does not modify the nature of elementary excitations in a Fermi system. Similarly to a Fermi gas, elementary excitations in a Fermi liquid are described by a certain momentum or quasimomentum, and appear in pairs: a particle (electron) and an antiparticle (hole); as in a Fermi gas, in the ground state a certain portion of the momentum or quasimomentum space is filled, and at low levels of excitation the particles and holes are generated in the vicinity of the Fermi surface.

In essence, a quasiparticle in a Fermi liquid describes the motion of an individual electron in the effective field of all other electrons. In particular, the charge of a quasiparticle is equal to that of an

electron, and its spin is also equal to that of electron, being  $1/2$ . Similarity between quasiparticles and true particles is especially clearly indicated by the fact that the number of quasiparticles in a Fermi system is equal to that of true particles.

The theory of Fermi liquid made it possible to calculate a number of characteristics of metals, such as specific heat, electric conductivity, magnetic susceptibility etc., i. e. to express them in terms of quantities which describe the properties of individual interacting quasiparticles: electrons and holes. Although we operate with quasiparticles instead of particles, interaction between the former ones cannot be neglected. It just is not small. Therefore we speak of a Fermi *liquid* of quasiparticles and not of a Fermi *gas*.

This is a singular situation. Indeed, the primary, or elementary object of the theory is not an individual noninteracting particle, as in atomic or molecular gases, or a quasiparticle, as a phonon in a crystal, but a quasiparticle interacting with other quasiparticles. It would seem that an object like this is just as complicated as the original interacting particles. Why then is it possible not only to derive comparatively simple formulas but also to construct a comparatively simple and lucid picture? The reason is this: the states of individual quasiparticles and particles are described in the same manner. Therefore all the descriptive images developed for interpreting the properties of a Fermi gas are applied to a Fermi liquid, while the possibility to use the states with a definite momentum or quasimomentum stems from the fact that these states are sufficiently long-lived if their energy is close to the Fermi energy. In order to restrict the analysis to only such states, the system must be in a state close to its ground state. For example, the temperature must be much lower than the degeneration

temperature. This is the main simplifying assumption of the theory of a Fermi liquid, and the one which makes the foundation of the theory.

### *More on electrons and holes*

When considering weakly excited states of an electron system, it is convenient to take its ground state for the reference (zero) level; for metals it means the quasi-momentum space partially filled by electrons. In this description particles and holes appear as elementary excitations of the electron system. Unfortunately, the term "hole" has one more meaning. Fig. 9 shows two

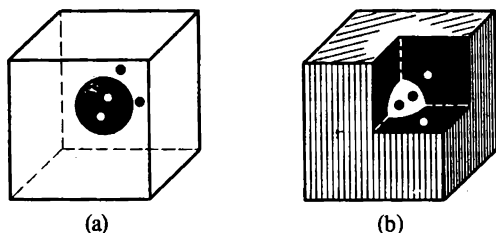


Fig. 9. (a) Electron Fermi surface, (b) hole Fermi surface. Both particles and holes (regarded as elementary excitations) are generated both in the former and the latter cases. In the case (a) the particles are outside the Fermi surface, while the holes are inside it. In the case (b) the situation is reversed. The sites in  $p$ -space free of electrons in the case (b) are also referred to as holes. Such holes are also found in the ground state of a metal

different situations. In Fig. 9a the states inside the Fermi surface are the states occupied by electrons, while in Fig. 9b the states inside the Fermi surface are the states free of electrons. The Fermi surface given in Fig. 9b is referred to as the hole surface, and the free

states in it are said to be *holes*. These holes, i. e. unoccupied sites in the band, are present even in the ground state. They behave as positively charged particles, at least in the case of a metal placed in a magnetic field. It is interesting to note that, in the case of a hole Fermi surface, the particles, i. e. electrons getting into the free sites in the band behave as holes in the former meaning of this too frequently used word.

## *Waves in electron gas*

Our habitual view that sound waves, i. e. waves of compression and rarefaction with frequency being inversely proportional to wavelength (see the section "Phonons in helium"), can propagate in any gas or liquid, is so strong that it is rather difficult to realize that sound waves *cannot propagate* in electron gases. Let us consider a uniform vibration. Uniformity in this case means that all electrons are shifted from their equilibrium position in phase, i. e. that the wavelength of vibration is infinitely long. Due to restoring electrostatic forces, the frequency of uniform vibrations is high. This frequency is called *plasma frequency*, since vibrations of electrons around heavy ions are characteristic not only for electron conductors but also for any electron-ion plasma. Squared plasma frequency is proportional to the number of electrons; plasma frequency  $\omega_L$  in high-conductivity metals is very high ( $\omega_L \approx 10^{15}$  1/s). In particular, it is much higher than the mean frequency of collisions for electrons. "From the standpoint" of plasma oscillations the electron gas is nearly free, although collisions must be taken into account when lifetime or damping coefficient of plasma oscillations is calculated. Obviously, uniform plasma oscillations constitute only a very crude model of wave



motions in an electron gas. A comparatively simple analysis enables to derive the relationship between the frequency and wavelength of plasma oscillations. And, of course, plasma oscillations can be described in terms of the corresponding quasiparticles. These are called *plasmons*. Plasmons are bosons. Plasmons do not exhaust the family of bosons describing the motions of electrons. We know that each electron is a tiny elementary magnet. Interaction between electron magnets makes possible oscillations in the electron system, so that specific waves can propagate through it. These waves are called *spin waves*, i. e. identically to oscillations of ordered magnetic moments of atoms in ferro-, ferri- and antiferromagnetics. The quasiparticles (again bosons) "corresponding" to these waves are called *magnons*. Spin oscillations are not accompanied by compressions and rarefactions in the electron gas, and therefore their frequency is not high. In metals, spin waves are detected in a metal's behaviour in alternating magnetic fields. It must be emphasized that the Fermi-liquid type of interaction between electrons is not really essential for the existence of plasmons, and they are present in electron gases. Spin waves, however, in non-magnetic metals exist only due to this Fermi-liquid interaction between electrons. They could not exist in metals if conduction electrons formed a gas.

Still, the spectrum of plasma oscillations of electrons is fairly poor if a strong external magnetic field is not applied. This is fully compensated, however, by the rich spectrum of weakly damped waves in electron conductors placed in sufficiently strong magnetic fields: helicons, magnetoplasma waves, cyclotron waves, and so forth. All these waves are macroscopic, i. e. their wavelength is much greater than interatomic distances. They were discovered "at the tip of the pen", but then were investigated experimentally. Recent years witnessed

the birth of a new branch of physics which studies a topic that was given a somewhat incorrect label of "solid state plasma". In essence, this branch is engaged in studying electron waves in solids placed in magnetic fields.

## *Excitons*

We have mentioned that interaction between electrons in semiconductors is not strong, so that Fermi-liquid effects in semiconductors are small. It is in semiconductors, however, that electron-electron interaction results in serious restructuring of energy spectra, namely in formation of *excitons*. Let us retrace our steps a bit. The simplest excitation in the electron system of a semiconductor consists in "transfer" of one electron to the conduction band, and formation of a hole in the valence band, the hole behaving as a positively charged particle. But if this is true, the hole and the electron must be attracted to each other. It is possible that they can form something similar to the hydrogen atom, or more precisely, positronium, because both the positive and the negative particles are light ones. This *quasi-atom* consisting of an electron and a hole is referred to as the *Vannier-Mott exciton*, or large-radius exciton. The motion related to transfer of excitation from one atom to another is referred to as the small-radius exciton, or the *Frenkel exciton*. The radius of the excited state is approximately equal to the size of the atom. Excitons are *really existing* quasiparticles. Their existence is revealed in optical properties of crystals. The energy of an exciton is smaller than the energy gap width, i. e. the distance between the conduction and valence bands. Exciton energy levels are found where no levels can be located from the standpoint of the

band theory. And since an exciton resembles a hydrogen atom, the system of exciton energy levels resembles that of hydrogen levels; this is indeed found out by the light absorption lines of some of the crystals (Fig. 10).

The structure of an exciton is well known. But though it does consist of an electron and a hole, the exciton must be treated as an *elementary*, i. e.

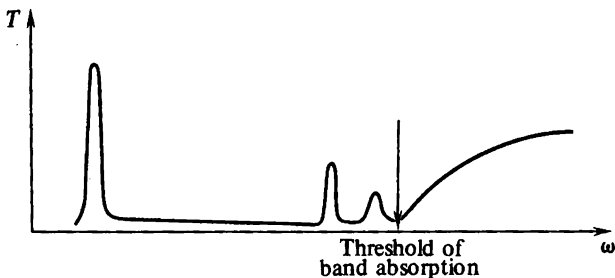


Fig. 10. Schematic presentation of the structure of a light absorption band in a crystal. Hydrogen-like absorption lines to the left of the threshold frequency point to the existence of excitons.

irreducible, quasiparticle. This means that the energy of motion of its components, viz. of the electron and the hole, is of the same order of magnitude as the energy of their interaction, while the energy of interaction between two excitons is small compared to the energy of each one of them. This justifies the conclusion that an exciton must be regarded as a distinct quasiparticle.

### *Polarons, fluctuations, etc.*

The roles played by electrons in solids depend on their position on the energy scale. Those at the deep

atomic levels are strongly bound to their nuclei and in fact do not respond to collectivization of ions into the crystal lattice. Not only do ions constitute the building material of crystals; each ion takes part in vibrations of the lattice, together with other ions, as a non-decomposable entity. But the electrons which were at the uppermost energy levels before atoms formed the crystal, i. e. valence electrons, break loose of their nuclei; their motion resembles that of free electrons, although they move in the force field produced by the ions. The properties of motion of such electrons are analyzed in detail by the band theory. We want to emphasize now that the band theory normally regards the ion lattice as something *static*, unmoving; all that it creates is the periodic distribution of forces. The possibility to separate the motion of electrons from that of ions arises not because of our desire to simplify the problem, but due to the physical reality: indeed, ion velocity is very small compared to that of fast electrons, and can be neglected in the first approximation. There are cases, however, of substantially more complicated character.

An "extra" electron in an ionic crystal slightly shifts the ions from their equilibrium positions due to its own electric field, i. e. it polarizes the lattice. In the absence of the electron such polarization would be unstable, but the displacement of atoms in the lattice results in a potential well for the electron (Fig. 11). The electron's energy is reduced and it is trapped into the well which it has "dug out" for itself. If this lowers the total energy, i. e. the energy of the electron plus that of the lattice, this complex entity will possess "the right to exist", thus producing a new quasiparticle, a *polaron*. The force applied to the electron in the potential well is transferred to the atoms of the lattice, and therefore the moving electron "drags

with itself" its well. Polarons may be charge carriers, although their motion is more hindered than that of free electrons in a band, because the motion must be accompanied by re-polarization of the surrounding medium. Therefore the effective mass (see below) of a polaron is substantially larger than that of an electron.

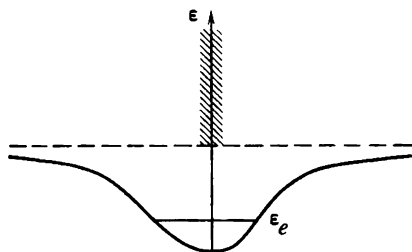


Fig. 11. An electron's energy,  $\epsilon_e$ , diminishes if the electron is trapped in a potential well. A polaron is an electron moving inside a potential well produced due to polarization of the lattice by the electron itself. A polaron can move as an individual entity

Polarons are not the only type of quasiparticles that "dig their own well" and then move with it. Similar entities were discovered in several types of solids during recent years, so far only on paper, i. e. as theoretical predictions or hypotheses. We will list some of them: *fluctuons* are generated when a fluctuation of one of the components of a disordered alloy is formed around an electron; this results in the energy well which traps the electron and thus may stabilize the fluctuation; *phasons* which are fluctuons accompanied by phase transition, such as a ferromagnetic region around an electron in a paramagnetic, etc.

Charged particles in liquid helium give examples of unusual fluctuons: a positive charge is surrounded by an "icicle", i. e. a region of solidified helium, while

a negative charge is surrounded by a spherical cavity which “houses” the electron. These entities are of substantial dimensions: about seven angström for an “icicle” and about twenty angström for a spherical cavity.

### *Half-way finish*

Our booklet started with an attempt to answer the question “What are the components of ...?”. And we came to the conclusion that the building material of a solid, its “building blocks”, is not identical to the “building matter” of motion in the solid. And although the energy of a solid body can be expressed as the sum of microscopic portions, or quanta which we agreed to call quasiparticles, the motion described by a quasiparticle involves, as a rule, the motion of a great number of, if not all, particles of the solid. Still, all *quasiparticles* are very similar to ordinary, but certainly quantum, *particles*. And indeed, the state of a free particle is fully determined if we fix its momentum, while the state of a quasiparticle is described by its quasimomentum. Both particles and quasiparticles are either bosons or fermions. Each particle possesses a set of properties characterizing its belonging to one of the classes of particles, such as protons, electrons,  $\pi$ -mesons, etc. Moreover, particles within each class are not merely identical, they are *indistinguishable*. The same is true of quasiparticles.

Note that in discussing particles and quasiparticles and listing their attributes, we did not mention their masses. Mass is not a necessary attribute of a particle. Photons and neutrinos have zero mass. Or rather, their rest mass is zero. Of course, we can always resort to Einstein’s formula  $E = mc^2$  and measure a particle’s

energy in mass units, or define mass as energy divided by  $c^2$ . When dealing with quasiparticles, it is standard to introduce the concept of “effective mass”, i. e. a quantity with dimension of mass, characterizing *dynamic* properties of a quasiparticle. A quasiparticle may behave differently in different conditions, so that “effective masses” proliferate. For the electron, for instance, one effective mass characterizes its motion in electric field, while another is used in magnetic field. Apparently, we could do without the effective mass, since the dependence of energy of a quasiparticle on quasimomentum, i. e. the dispersion law, is a complete characteristic of its dynamic properties. Effective mass is introduced for the sake of convenience. It is expressed in terms of the dispersion law. Here is one of the most frequently employed definitions of the effective

mass:  $m_{\text{eff}}^{-1} = \left( \frac{\partial^2 \epsilon}{\partial p^2} \right)$ . If we know that one group of

electrons possesses much smaller effective mass than that of the other group, we immediately realize that the electrons of the first group are more mobile. Consequently, they are the charge carriers that make maximum contribution into conduction in the metal or semiconductor.

We have emphasized several times that one of the main features of the elementary type of motion, namely of the possibility of introducing quasiparticles, is the additivity of the contributions of quasiparticles into the energy of a solid. If a solid's energy is measured off the ground state, it is the sum of energies of quasiparticles. And although this statement is the dominant idea of the concept of quasiparticles (the reader recognizes that we try to construct a gas situation), it is far from being rigorous. A solid's energy is only *approximately* equal to the sum of energies of quasi-

particles. This is a real-gas situation all over. Indeed, there are no truly ideal gases in nature. Gas particles (molecules or atoms) interact with one another even in very rarefied gases. This is revealed in many properties of gases: in specific heat value, in the real gas equation of state differing from that of the ideal gas, in the actual values of the coefficients of diffusion, heat conduction, etc., and so forth.

### *Colliding quasiparticles*

In a way, the last paragraph summarizes the result of interaction between atoms (or molecules) of a gas. This is a macroscopic level. A gas is a macroscopic ensemble of atomic particles and it demonstrates the fact of interaction between atoms and molecules it contains. But how is the interaction realized? What happens to atomic particles in the course of interaction? The atoms and the molecules *collide* with one another. Collisions would not happen in a truly ideal gas, and each particle would follow a straight-line path until its impact with the wall of the vessel. A mathematical description of the collision process is possible if *forces* between particles and *structure* of the particles are known. We have already indicated at the beginning of the book that a gas molecule can be considered elementary in some cases and cannot in others. If the inner structure of a molecule or atom can be completely neglected, a collision between two particles can only result in redistribution of the momentum of translational motion of the particles. The laws of energy and momentum conservation are, naturally, satisfied: energy and momentum of the translational motion of the particles prior to collision are equal to energy and momentum of the particles after it. Such collisions are called elastic.



Collisions are said to be inelastic if they result in something happening to one or both of the colliding particles, for example, if an atom switches from its ground state into the excited state. No doubt, both energy and momentum are conserved in inelastic collisions as well. One must only take into account what was "spent" on restructuring the particle. Sometimes collisions may lead to appearance or disappearance of particles. For instance, two colliding hydrogen atoms may completely disappear, producing a new particle, viz. a hydrogen molecule  $H_2$ . Or a collision of two atoms may ionize one of them, separating an electron. The atom disappears, and two new particles are produced: an ion and an electron.

"Dramatic" collisions, in which particles perish and appear, are possible even in the realm of "truly" elementary particles (electrons, nucleons, mesons, etc.). In specific conditions, a collision of a positron and an electron results in their disappearance (annihilation) and production of photons. And sometimes no collision is necessary for disappearance of a particle: a neutron in a free state does not live eternally but decays into a proton, an electron, and a neutrino. These examples could be multiplied, since the number of unstable particles, i. e. those with finite lifetimes, is much greater than that of the stable ones. There are only four types of stable particles: a proton, an electron, and two sorts of neutrino. The number is doubled if we take into account their antiparticle counterparts.

This long digression on collisions of particles was required for the sake of a comparison, because quasiparticles undergo collisions, too. What then is the difference between their collisions and those of ordinary particles? The fundamental difference lies in the *conservation laws*. Energy is always conserved in the

collisions of quasiparticles, while quasimomentum may not. Those collisions in which quasimomentum is conserved are said to be *normal*, while those which violate conservation of quasimomentum are referred to as *umklapp processes*. It does not mean, though, that quasimomentum is “scot-free” in umklapp. Even if quasimomentum is not conserved, it changes by a quite *definite* quantity, equal to one or several periods of the lattice in the quasimomentum space. If we identify quasimomentum with ordinary momentum, it is convenient to picture the crystal as something whole which can absorb momentum by definite portions.

Another, rather less fundamental but certainly very substantial difference typical of collisions of quasiparticles consists in the following. Transformations of elementary particles are observed when participants of the collision have high energies, and their velocities approach that of light. Therefore such processes are studied by the *relativistic quantum mechanics*. Transformation of quasiparticles, however, is a “run-of-the-mill” affair, taking place in all solids: a phonon of one type decays into a pair of phonons of a different type. A phonon in ferromagnetics may decay into a pair of magnons, and an electron in a metal may absorb or emit a phonon. The fact that a crystal in its ground state contains no quasiparticles signifies that no quasiparticle is eternal. Lifetimes of quasiparticles are very different and substantially depend on temperature of the solid: the higher the temperature, the shorter the lifetime. This is understandable since lifetime of a quasiparticle in a given state depends on the frequency of its collisions with other quasiparticles; and the higher the temperature, the greater the number of quasiparticles. At sufficiently high temperatures quasiparticles collide so often that they do not have enough time to reveal their identity between two collisions. Quasiparticles no more play the

role of elementary excitations, not being applicable to describing the properties of solids. The lower the temperature of a solid, the lower the frequency of collisions, the more individual is the life of each quasiparticle; using more rigorous terms, the more ideal is the gas of quasiparticles. This is why the concept of quasiparticles is mostly used to describe properties of solids at low temperatures.

As interaction of ordinary particles, interaction of quasiparticles (their collisions) changes the values of equilibrium characteristics of the gas of quasiparticles as well as characteristics of a solid: specific heat, thermal expansion coefficient, etc. But the most important role played by collisions is that they are the reason for relaxation processes in solids. Were it not for collisions between quasiparticles, the gas of quasiparticles could never reach equilibrium; besides, a solid could not come into equilibrium with the ambient medium.

The relaxation, i. e. the process of equilibrium onset, is not the only process in a solid which proceeds via collisions of quasiparticles. All the kinetic processes and phenomena, among them electric and thermal conduction, resonance phenomena, frequency dispersion of a number of dynamic susceptibilities, and many others, are essentially the result of collisions between quasiparticles. The formulas describing kinetic processes, and all the kinetic coefficients include, as an attribute *sine qua non*, a quantity characterizing the process of collision of the quasiparticles. Most often this is the *free path length*, i. e. the mean distance which a quasiparticle travels between two consecutive collisions, or the mean distance that the quasiparticle can travel, since disappearance of a quasiparticle is a very common outcome of a collision. This does not necessarily mean that the number of quasiparticles diminishes. Collisions in which quasiparticles are produced may be just as

frequent. Investigation of kinetic processes is a method of studying quasiparticles, and offers an inexhaustible well for the efforts of both experimenters and theorists.

Concluding this section, we want to underline that the gas philosophy which we were trying to follow is as a result substantially extended. The free path length is a typically gas quantity. Indeed, this concept can be used when and only when the process of collision has a much shorter duration than the free path length of a particle, i. e. when a particle between collisions feels free *for a long time*. This is essentially a definition of a gas as a state of aggregation of matter.

### *Additional complications*

Let us select for investigation a comparatively simple object and discuss its dynamical properties at all temperatures at which this object exists. To be frank, it is quite a problem to find a solid which would satisfy our goals in all respects; therefore we shall restrict ourselves to the consideration of a *nominal* solid, to which all the required properties are attributed. To begin with, we take up the case of an insulator (dielectric) in which excitations of the electron system, i. e. production of an electron-hole pair, have negligible probability and can be ignored altogether. More precisely, this means that the minimum energy of the pair production is much higher than the melting point of a solid. Such solids are numerous, diamond being one of them. The possibility to neglect excitation of the electron system entitles us to regard atoms, molecules or ions as truly elementary particles: not only as the building material of "our" solid, but also as the particles responsible for its *dynamic* properties. The motion of

the particles is their vibrations around equilibrium positions. In quantum terms this motion is described by introducing special quasiparticles — phonons (see the section “Phonons”).

We thus see that the desire to discuss the dynamic properties of solids in a wide range of temperatures entails the necessity to analyze once again the properties of the gas of phonons, although it has already been mentioned that the quantum (“phonon”) terminology is not very convenient at high temperatures\*.

As all quasiparticles, phonons are characterized by a specific range of energy; it is customary to speak simply of the characteristic energy of quasiparticles of a given type. In the case of phonons it is referred to as the Debye energy, or the Debye temperature, the two differing by a factor  $k = 1.4 \cdot 10^{-16}$  erg/deg. Let us denote it by the letter  $\theta$ . The physical meaning of the Debye energy is simple enough: it is the maximum energy of the phonon, i. e. the maximum frequency of atoms' vibrations, times Planck's constant:  $\theta = \hbar\omega_{\max}$ .

We have mentioned in the section “Phonons” that even at the melting point the vibration amplitudes  $u$  of the atoms are small compared to interatomic distance  $a$ . The Debye energy  $\theta$  is much less than the bonding energy of the atoms in crystals,  $\varepsilon_0$ ; the order of magnitude of  $\varepsilon_0$  can be found from the melting point ( $T_{\text{mel}}$ ):  $\varepsilon_0 \approx kT_{\text{mel}}$ . Let us try to understand what causes the small value of  $\theta$ . If the bonding energy of two neighbour atoms is  $\varepsilon_0$  then the frequency of vibrations of the atoms in respect to one another is  $\omega_{\max} \approx$

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\* This section will draw attention not only to advantages of the theory but to its drawbacks as well. This should not make the reader upset: realization of the shortcomings is the first step to their elimination.

$\approx \sqrt{\epsilon_0/Ma^2}$ , where  $M$  is the mass of one atom. Hence,  $\theta \approx \hbar\sqrt{\epsilon_0/Ma^2}$ . We could, of course, take the values of the parameters in the last formula from a corresponding reference book and ascertain that  $\theta \ll \epsilon_0$ . This will not add to our understanding, though. Another approach can be chosen, viz. to estimate the "complicated" parameters  $a$  and  $\epsilon_0$  in the formula and then try to interpret the result. Here is one possibility. Interatomic spacing  $a$  is approximately equal to atomic size, determined by the quantum motion of electrons around the nucleus. Therefore,

$$\frac{mv^2}{a} = \frac{e^2}{a^2}, \quad v \approx \frac{\hbar}{ma},$$

$m$  standing for the electron mass. The first of the formulas is classical and the second is quantum, stating that an atom's diameter is equal to the de Broglie wavelength of the electron. It is readily found that these two formulas yield a reasonable, by the order of magnitude, value for the atom's diameter:

$$a = \frac{\hbar^2}{me^2} \approx 4 \cdot 10^{-8} \text{ cm.}$$

On the other hand, the bonding energy of the atoms is of the order of  $e^2/a$ . In order to simplify the arguments, we assumed the solid to consist of ions. Combining all the above formulas into one, we readily obtain

$$\theta = \epsilon_0 \sqrt{m/M}.$$

The reason for the low value of the Debye energy is now quite clear: the atom's mass is much greater than

that of the electron ( $m \ll M$ ). Since  $\theta \ll \varepsilon_0$ , we can investigate the properties of a solid both at the temperatures small compared to the Debye temperature, and at  $T$  much greater than it.

The Debye temperature plays a very important role in the solid state physics, separating the quantum region ( $T < \theta$ ) from the classical region ( $T > \theta$ ). And more precisely, at low temperatures ( $T < \theta$ ) quantum approach is required. This is caused by the fact that the classical equipartition law, requiring that the mean energy per each degree of freedom must equal  $kT$ , leads to contradiction with quantum mechanics at  $T < \theta$ : the energy of an oscillator with frequency  $\omega$  cannot be less than  $\hbar\omega$ .

Let us begin with the classical high-temperature range  $T \gg \theta$ . According to the equipartition law the internal thermal energy of a solid is  $3NT$ . What is the meaning of this result "in the phonon language"? It means that the number of phonons is  $\approx 3NT/\langle \hbar\omega \rangle$ . The symbol  $\langle \hbar\omega \rangle$  denotes the mean energy of a phonon. And since  $\langle \hbar\omega \rangle < \hbar\omega_{\max}$ , plus  $T \gg \theta$ , it becomes clear that the number of phonons in the classical region is very high, much greater than the number of atoms.

There is nothing paradoxical in this statement: *each* oscillator "simply" occupies a state described by the *first (principal)* quantum number. Its energy is  $E_n = (n + 1/2)\hbar\omega$ ,  $n$  being an integer number, with  $\langle n \rangle \gg 1$ . This feature makes description in phonon terms not really convenient: the frequency of phonon-phonon collisions is high, and lifetimes of individual phonons are short; a phonon is not given enough time to reveal its identity to the full. "Atomic language", or "site" representation, emphasizing the fact that each atom vibrates around "its own" site in the crystal lattice, is more convenient because the amplitude of atomic

vibrations  $u$  remains small compared to interatomic spacing even at  $T \gg \theta$  \*.

The "phonon language" is not at odds with the site language. It can be proved that short-wave phonons, dominating at  $T \gg \theta$ , provide correct description of the almost non-correlated vibrations of the atoms.

Hence, the energy of phonons at high temperature is proportional to temperature. This statement is not quite correct, since quantum corrections which are indeed small at high temperatures result in a steeper decrease of phonon energy with decreasing temperature than predicted by the linear law. At temperatures close to and below the Debye temperature, the classical linear law vanishes altogether and is replaced by the quantum law. In the vicinity of the absolute zero the energy is proportional to the fourth power of temperature

$\left( E_{\text{ph}} \sim N \frac{T^4}{\theta^3} \right)$ . Fig. 12 plots phonon energy as a

function of temperature in a wide temperature range, and marks the Debye temperature  $\theta$ , in the vicinity of which the classical law  $E_{\text{ph}} \sim T$  changes to quantum dependence  $E_{\text{ph}} \sim T^4$ . Note that in the temperature range close to the Debye temperature a phonon gas possesses all the specific properties of a quantum gas: non-force nature of interaction results in correlated

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\* Vibration amplitude is less than the average interatomic spacing even close to the melting point. A crystal melts not because its atoms "mix" due to thermal vibrations. The liquid phase is more "advantageous" energy-wise at high temperatures than the crystalline phase, since the free energy of the liquid phase is less than that of the crystalline phase. The vibrational motion is still maintained in the liquid, again with small vibration amplitude; the atoms vibrate, however, around random, uncorrelated with one another, equilibrium positions. We are not going to touch upon the liquid phase in this booklet.



motion of phonons, and the length on which this correlation is appreciable coincides with the mean wavelength of the phonons, decreasing with the rise in temperature.

The above-outlined simple dependence of energy on temperature is practically never observed in real solids. Complications are caused by several factors.

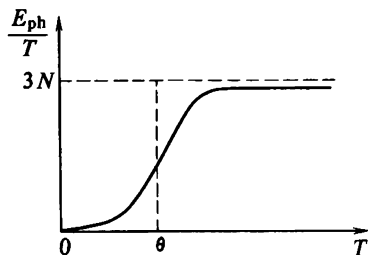


Fig. 12. Energy of a phonon gas is a function of temperature. The curve is idealized. In actual solids the energy of the thermal motion of atoms is a more complicated function of temperature. The dependence in the vicinity of  $T=0^\circ\text{K}$ , viz.  $E_{ph}/T \sim T^3$ , is valid for all solids

Phonons do not exhaust all the types of motion in solids. True, this fact could be neglected, since it is possible to single out the phonon fraction of the internal energy of a solid. Another fact is even less pleasant. The above-mentioned laws ( $E_{ph} \sim T$  at  $T \gg \theta$  and  $E_{ph} \sim T^4$  at  $T \ll \theta$ ) are indeed valid for *all* solids and are essentially non-sensitive to the specifics of interatomic interactions in crystals. The first law, i. e. the classical one, is based exclusively on the equipartition law, while the second, quantum, law is based on the fact that the lowest-energy phonons which play the main part at low temperatures are nothing but quanta of a sound, and as such must

have frequency *inversely proportional to wavelength*. Such waves can propagate in any solid, and therefore the phonon fraction of the internal energy  $E_{\text{ph}}$  is proportional to  $T^4$  at sufficiently low temperatures in all solids. In the intermediate temperature range at  $T \sim \theta$ , however, the dependence of phonon energy on temperature is substantially defined by the specific properties of phonons, i. e. by the specific dependence of energy (frequency) of a particular phonon on its quasimomentum; in other words, by the dispersion law. The dispersion law can be calculated knowing the forces between the atoms, and from this law we can find the energy of the phonon gas. The opposite course may be used: one could try to determine the energy spectrum of phonons from the dependence  $E_{\text{ph}}(T)$ .

When phonons are already numerous ( $T \sim \theta$ ) but their collisions are not yet too frequent, the phonon approach is useful, at least in principle. The dynamic properties of solids can be formulated in "phonon terms", i. e. the macroscopic quantities can be expressed in terms of phonon characteristics: phonon dispersion law, free path length, etc. Unfortunately, the calculations are very complicated, and this intricate nature of mathematical calculations often blurs the lucidity of ideas. Nevertheless, application of the laws governing the motion and collisions of phonons yields very accurate approximation of the temperature dependence of the energy of the solid (its phonon fraction, to be precise) and of some other characteristics.

Not always, though... Among the events taking place in solids there are some that in a macroscopic world should be considered catastrophes: the structure of a crystal undergoes a jump-wise change as a result of thermal motion of the atoms or due to some external factor acting on the solid. This event is called a phase transition. Examples of phase transitions

are melting of crystals or solidification of the liquid. There are, however, less cardinal transformations. For instance, below  $1539^{\circ}\text{C}$  iron is crystalline. But as temperature changes, the structure of its lattice undergoes modifications: a body-centered cubic lattice, or  $\alpha$ -phase, is stable at low temperatures ( $T < 910^{\circ}\text{C}$ ), a face-centered cubic lattice, or  $\gamma$ -phase, is stable if  $910^{\circ}\text{C} < T < 1400^{\circ}\text{C}$ , and a body-centered lattice, or  $\delta$ -phase, is again stable at  $1400^{\circ}\text{C} < T < 1539^{\circ}\text{C}$ .

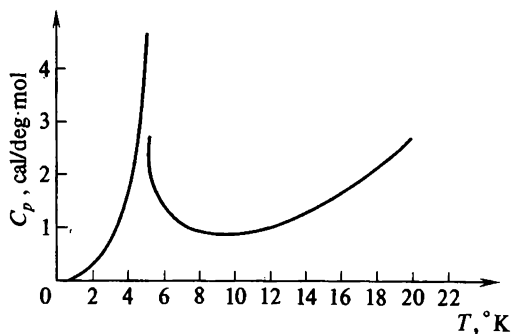


Fig. 13. Specific heat,  $C_p$ , of  $\text{NiCl} \cdot 6\text{H}_2\text{O}$  crystals as a function of temperature in the vicinity of the point of transition from the ferromagnetic (high-temperature) to antiferromagnetic phase.  $C_p$  tends to infinity at the transition point

As a rule, phase transitions, i. e. restructuring of crystals, cannot be described in terms of quasiparticles. Temperature dependences of quantities characterizing various properties of crystals may reveal jumps, kinks and sometimes "infinities", when in the transition point one of the characteristics of the solid tends to infinity (Fig. 13). This fact was the main reason for choosing the heading of this section: "Additional complications".

## *Digression on phase transitions*

It is not easy to introduce quasiparticles correctly, to find out the laws governing their motion, to investigate the properties of quantum gases of quasiparticles, to take into account the collision-induced corrections, to calculate by using the "quasiparticle" approach the various characteristics of solids as functions of temperature, frequency of the electromagnetic field, or parameters of the external factors; but this is not all. Far from it . . . . We have already mentioned that in certain situations quasiparticles "refuse to work". As a rule, quasiparticles are not conducive to understanding the nature of phase transitions, i. e. transformations taking place in solids, or to determining the laws regulating the behaviour of solids close to the phase transition point. It is often possible, however, to determine how the phase transition affects quasiparticles or, to be more precise, how it modifies the energy structure of the solid. We could not go into the theory of phase transitions here. This is a separate, interesting field of solid state physics, and which is now very actively progressing. The reader will have to take for granted that there are different types of phase transitions. But then they must be enumerated: phase transitions of the first kind, those of the second kind . . . . Phase transitions of the first kind are simpler, due to a simple cause of the transition. The reason is clear from Fig. 14. It consists in the tendency of every system to occupy the least-energy state\*. The "advantageous" phase is *A* above the

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\* To be precise, a solid placed at a temperature distinct from zero tends not to the state of minimum energy but to the state of minimum *free* energy. The difference between the free energy and "just energy" can be illustrated as follows: in two states of a solid with identical energy but different degree of ordering the free energy of the less ordered state is the smaller of the two.

transition point ( $T > T_0$  in Fig. 14), and  $B$  below  $T_0$ . The point  $T_0$  has nothing special either for the  $A$ - or  $B$ -phase. Were the  $A$ -phase alone, nothing would happen to it at  $T_0$ . And as a rule, nothing happens:  $A$ -phase can be supercooled, and  $B$ -phase superheated. In both these phases the solid possesses an energy spectrum, i. e. a set of quasiparticles specific for each phase, and enabling to calculate many properties of the  $A$ - and  $B$ -phases; in particular, one could find the

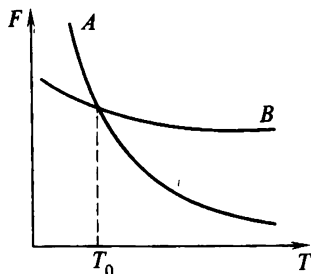


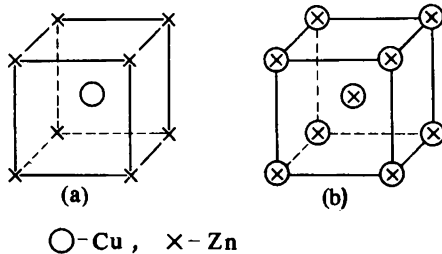
Fig. 14. The nature of any phase transition of the first kind: free energies of the two phases coincide at the transition point. The  $B$ -phase is favoured at  $T < T_0$  ( $F_A > F_B$ ), and the  $A$ -phase is favoured at  $T > T_0$  ( $F_A < F_B$ ). The point  $T_0$  possesses no special properties "from the standpoint of any of the two phases". Consequently, both superheating and supercooling are possible

properties of the transition itself, for instance, calculate the amount of heat liberated or absorbed in the process of the transition. And the contribution of quasiparticles must of course be taken into account when determining the "advantages" of the phases, i. e. calculating the free energy in the vicinity of  $T = T_0$ .

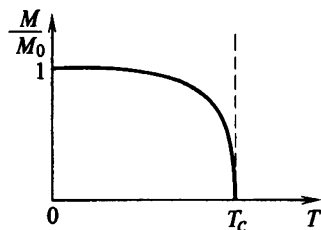
The situation is much more complicated in the case of phase transitions of the second kind. The examples of the phase transitions of the second kind are numerous.

They include the transition from the normal state of a metal with finite electric resistance to the superconducting state with zero resistance, the transition from the paramagnetic to the ferro- or antiferromagnetic state in which magnetic moments of atoms are arranged in a certain order, and ordering in alloys. In the last example the physical significance of the transition is clearly demonstrated in Fig. 15. The feature characteristic for all these and many other cases lies in the fact that in the transition point itself, i. e. at  $T = T_{\text{crit}}$ , both phases coincide: the transition transforms the  $A$ -phase into  $B$  or  $B$  into  $A$  in a continuous manner, but *only one* of the two phases can exist at any  $T \neq T_{\text{crit}}$ , the other being absolutely unstable. The "old" phase is gradually getting "prepared" to the transition, which is realized at  $T = T_{\text{crit}}$ . Although phase transitions of the second kind are more smooth than those of the first kind, and this statement can be formulated rigorously (it is illustrated in Fig. 16), the transition itself nevertheless involves a "jump" in the sense that there appears something that did not exist before, for example, a spontaneous magnetic moment of a ferromagnetic, or the density of the superliquid component in helium, or something else. It is important that this "something" does not yet exist in the transition point, where it equals zero, but it gradually increases away from the transition point on one side of it, as a rule on the low-temperature side. The apparent manifestation of the complex nature of the second-kind phase transitions is tending to infinity of a number of physical characteristics of the solid (various susceptibilities, specific heat, etc.) and anomalous effects of fluctuations in the motion of atomic particles.

Let us try to outline the situation by taking up a specific example, namely the transition from the paramagnetic to the ferromagnetic state. Or, resorting to



*Fig. 15.* The structure of an ordered copper-zinc alloy. The alloy is disordered at  $T > T_{\text{crit}}$ : the probabilities of finding in a lattice site an atom of copper or zinc are equal (a). At  $T < T_{\text{crit}}$  copper atoms have higher probability to be located at certain lattice sites, and zinc atoms – at other sites. The positions of both copper and zinc atoms are unambiguously determined at the absolute zero (b). The phase transition from the disordered to ordered state is a phase transition of the second kind provided the structure changes smoothly at the transition point (for temperature approaching  $T_{\text{crit}}$  from lower temperatures)



*Fig. 16.* Typical temperature dependence of the spontaneous magnetic moment of a ferromagnetic at temperature below the Curie temperature. The magnetic moment drops to zero at the Curie temperature ( $T_{\text{crit}} = T_c$ ) ( $M_0$  denotes the value of the magnetic moment at  $T = 0^\circ\text{K}$ )

a more pompous phrase, let us investigate the behaviour of a system of magnetic moments close to the transition point  $T_{\text{crit}}$ , which in this case is referred to as the Curie temperature. At high temperatures the energy of interaction between magnetic moments can be neglected, each magnetic moment "leads an independent life", and there is practically no correlation in the motion of spins. Although magnetic moments are fixed to atoms which are themselves fixed in lattice sites, it is convenient to picture them as a gas (often referred to as a lattice gas). At low temperatures, i. e. close to the absolute zero, all the magnetic moments are ordered (for instance, all of them point in the same direction) and the motion is realized in the form of spin waves, i. e. magnons (see the section on magnons). From the "standpoint" of magnons, the solid is again a gas, viz. a gas of magnons. There is almost no correlation in the motion of magnons.

Therefore, the two considered limiting cases far from the transition temperature led us to a simple gas situation. The thermal motion in a macroscopic system is realized in the form of uncorrelated motion of microscopic "elementary" particles or quasiparticles. These are magnetic moments at high temperatures, and spin waves, or magnons, at low temperatures.

Both these pictures are incorrect at temperatures close to the transition point. Magnetic moments are not independent any more, while spin waves cannot be introduced yet. Gas approximation is not applicable yet at  $T \approx T_{\text{crit}}$ . In this case the interaction between particles, represented by magnetic moments or magnons in the example considered, becomes especially significant and results in the unavoidable correlation of motion of microscopic particles. The closer to the transition point, the higher the number of particles involved in the



interaction; in other words, the radius of correlation tends to infinity as the Curie temperature is approached. It is exactly this feature that causes the strange properties and complications in phase transitions of the second kind, such as tendency to infinity in a number of properties, etc. The same reason causes the special interest of the physicists to the behaviour of macroscopic systems in the vicinity of the phase transition points of the second kind. Indeed, a condensed system reveals the dynamic *properties of a system of interacting particles* in full, "with no allowances made", only in such conditions. In the ranges far from  $T_{\text{crit}}$  we can always, however intricate the method, rephrase the analysis of properties of interacting particles into that of *weakly* interacting particles or quasiparticles. The complexities in the behaviour of particles in the vicinity of a second-kind phase transition point become especially lucid if we compare the increase in the correlation radius with the fact that this increase is *in no way related* to the increase in the radius of *force* interaction. The character of force interaction, i. e. its intensity, dependence upon the distance between particles, etc., is as a rule almost independent of temperature. Unfortunately, it is very difficult to explain the nature of increase in the correlation radius without resorting to fairly complicated arguments.

We have described a phase transition of the second kind from a rather bizarre standpoint. In many cases the increase in the correlation radius at  $T \rightarrow T_{\text{crit}}$  takes place in such a narrow temperature range around the transition point, that it is practically unobservable; in such cases an experiment demonstrates simplified properties of a solid close to  $T_{\text{crit}}$ , when, for instance, specific heat appears to undergo a finite jump but does not tend to infinity. An especially typical example is provided, in this sense, by the transition to the superconducting

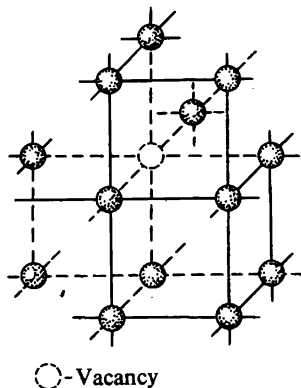
state. The anomalies due to the increase in the correlation radius must be observed in a temperature interval so narrow that it renders them practically undetectable. As a result, if these “fine details” are totally ignored, the concept of quasiparticles can be used up to  $T = T_{\text{crit}}$ . By returning to ferromagnetics, we can illustrate the above argumentation by the following example: each magnon “corresponds” to one disordered magnetic moment. When the number of magnons equals that of magnetic moments in a solid, all its magnetic moments are disordered. And as the number of magnons increases with increasing temperature, the above considerations enable to calculate the Curie temperature and all the properties of the transition in terms of magnon characteristics. Although this method of computation is not rigorously substantiated, the obtained values for most of the quantities do not differ too much from the true values.

### *Quantum crystals. Quantum diffusion. Vacancies*

In the preceding chapters we treated a crystal at absolute zero temperature, devoid of thermal motion of atoms, not so much as physical object but rather as a *region of existence* of quasiparticles, a “habitat” in which quasiparticles “lived their lives”: collided and transformed, merged and decayed. It was in quasiparticles that quantum features of dynamics, both individual and collective, were apparent. Obviously, these quantum peculiarities were reflected in the properties of solids, for instance, in the dependence of specific heat of crystals on temperature close to the absolute zero. Therefore each crystal could be called a quantum crystal. The fate of this term, “quantum crystal”,

however, is different. It was introduced into the scientific parlance rather recently and was assigned a quite definite meaning which does not coincide with that wide interpretation that was presented above.

We prefer to start not with quantum crystals, but with quantum diffusion, which is interesting in itself but at the same time is handy as a "seed crystal" for elucidation of what a quantum crystal is. To simplify the matter, we shall discuss self-diffusion, i. e. random wandering in the crystal of the atoms which constitute this crystal. How can this wandering be realized? Here is the most natural model. Each crystal, even be it the most perfect, contains vacancies, i. e. unoccupied sites of the crystalline lattice (Fig. 17). An atom neighbouring



*Fig. 17.* A crystal with a vacancy, i. e. an unoccupied lattice site. Any of the atoms on sites neighbouring the vacancy may occupy its site. This will shift the vacancy by one lattice constant

the vacancy can jump onto the vacant site, another atom jumps onto the thus vacated site, liberating the site for the next jump of the atom whose motion we consider. Note that the whole sequence of jumps of the atoms can be described as motion of a vacancy, and that this motion of a vacancy can be considered as an elementary act

of self-diffusion. What is then the mechanism by which a vacancy moves, and why does a crystal always contain vacancies? Let us start with the second question. A vacancy is formed when an atom moves from inside the crystal to its surface; this requires that certain energy be expended, thus making the formation of a vacancy disadvantageous, energy-wise. But since a solid tends at a temperature distinct from the absolute zero not to the least energy but to the least free energy (see the footnote on p. 73), a solid may lose something in energy increasing at the same time the degree of disordering. The competition between the gain in energy and the increase of disorder minimizes the free energy in a solid at non-zero temperature when it contains certain concentration of vacancies. The same idea is expressed by stating that "atoms of vacuum", in other words a gas of vacancies, are dissolved in a solid. The motion of vacancies at a finite temperature ( $T \neq 0$ ) is also readily modelled. As a result of fluctuation in the thermal motion of an atom neighbouring the vacancy, the atom "clears" the potential barrier separating it from the unoccupied site (Fig. 18). These concepts, which were formulated by Ya. I. Frenkel already at the beginning of this century, explain the diffusion process not only qualitatively but enable to calculate the diffusion coefficient; in particular, it is shown that the diffusion coefficient decreases exponentially when temperature diminishes. Lowering of temperature makes formation and motion of vacancies more difficult.

The above presentation of self-diffusion operated exclusively with classical and not quantum concepts. The quantum-mechanical wave properties of atoms permit another mechanism of vacancy motion; an atom may cross a potential barrier not by a fluctuation-caused "jump" but due to the effect of tunneling, i. e. a

quantum possibility to pass under (pass through should be a better term) the potential barrier. Comparison of the two mechanisms demonstrates that, firstly, the classical above-barrier mechanism is always more probable at high temperatures, and secondly, that the

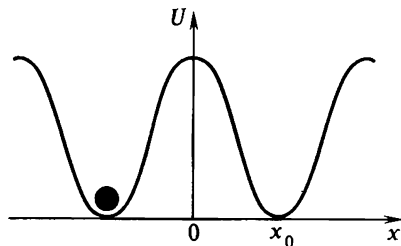


Fig. 18. Potential energy of an atom neighbouring a vacant site ( $x_0$ ). In order to get to the unoccupied site of the lattice, an atom must climb the potential barrier  $U(0)$

probability of the under-barrier tunneling mechanism of motion is the higher the lighter the atom. In the case of sufficiently heavy atoms the probability of tunneling is so small that the role of quantum effects in diffusion becomes negligible. Let us formulate it: *the diffusion is called quantum if the main role in the site-to-site motion of atoms is played by tunneling and not by the above-barrier transition of atoms from one to another equilibrium position.* The transition from the classical to quantum diffusion is manifested in the deviation of the “habitual” temperature dependence of the diffusion coefficient (Fig. 19).

Let us consider now the quantum crystal. An attentive reader may have noticed certain inconsistency in our reasoning. Quantum self-diffusion needs vacancies. But how can they be produced in an equilibrium

crystal at the absolute zero if their existence is disadvantageous, energy-wise? It can be shown that the possibility for a vacancy to tunnel explains the fact of the presence of vacancies in an equilibrium crystal at  $T = 0^\circ\text{K}$ . Look at Fig. 20. Its left-hand side illustrates

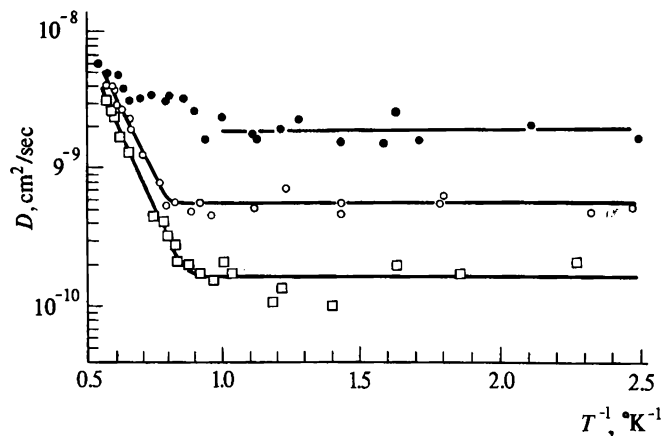
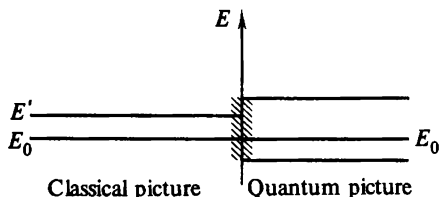


Fig. 19. Temperature dependence of the diffusion coefficient,  $D$ , of  $\text{He}^3$  atoms (at various concentrations of  $\text{He}^3$ ) in solid  $\text{He}^4$ . As given by the classical laws, the diffusion coefficient at high temperatures ( $T > 1^\circ\text{K}$ ) rises exponentially as temperature increases; the shape of the curves at  $T < 1^\circ\text{K}$  indicates that quantum diffusion sets on

the following statement: the energy of a crystal with a vacancy ( $E'$ ) is greater than that of a crystal without vacancies ( $E_0$ ). The right-hand side calls for an explanation. "From the standpoint of a vacancy" a crystal is a periodic field of forces, in which the possibility for a vacancy to move is caused by its tunneling through the barriers separating one lattice site from its neighbours. But we have already found that the

motion in the periodic field resembles the motion in the free space. In particular, the state of a vacancy must be described by a certain quasimomentum, i. e. a vector with properties similar to momentum. Moreover, a vacancy (to be precise, crystal with vacancy) may



*Fig. 20.* Energy of a crystal with a vacancy,  $E'$ , in the framework of classical concepts is always greater than that of a crystal without a vacancy. The right-hand side of the diagram shows the band of permitted values of energy in a crystal with a vacancy. One can see that in the framework of quantum concepts a vacancy-containing crystal may have energy below that of an ideal crystal

have different energies, depending on the value of quasimomentum of the vacancy. The quantum approach demonstrated that the energy of a crystal with a vacancy occupies a band. If the bottom of this band is below  $E_0$  (see the right-hand side of Fig. 20), then the formation of a mobile vacancy is advantageous from the standpoint of energy, and vacancies must be present in equilibrium crystals at  $T = 0^\circ\text{K}$ . But look at the changes caused by quantum mechanics in our concept of the vacancy. It was an empty site in the lattice that could be filled by a neighbour atom, and now it turned into a true quasiparticle with all the proper attributes, viz. quasimomentum, a band of allowed values of energy, dispersion law, and so forth. The quantum vacancy deserves a special name, and one was assigned to it.

A quasiparticle describing the motion of a quantum vacancy is referred to as a vacancion.

A light impurity atom in a crystal also behaves as a quasiparticle. Such quasiparticles are called (so far only half-jokingly) impuritons. Both self-diffusion and diffusion can be described in the quantum region as gas diffusion of vacancions and impuritons, with application of the concepts of the kinetic theory of gases, such as free path length, scattering cross section, etc. Another important factor taken into account by the theory of quantum crystals is the fact that as a result of tunneling a vacancy or an impurity atom gain the properties of particles moving in a *periodic* field. However paradoxical it may sound, a single tunneling event is much less probable than the quasi-free motion with energy belonging to the band of allowed values (see the section "Electrons. Energy bands").

How can we describe a crystal containing vacancions? If we could take photographs which make it possible to see each atom of the crystal, we would find a strange situation: a sequence of photographs separated by time intervals in which one and the same site in the lattice would sometimes contain an atom and sometimes not. A more rigorous statement, although not so demonstrative, is: *positions of atoms in a crystal are not strictly defined*. Atoms are mobile because not all lattice sites are occupied (at  $T=0^\circ\text{K}!$ ). The term "quantum crystal" was given exactly to a crystal with vacancions. The quantum crystal is a special state of matter, occupying an intermediate position between the quantum liquid and the classical crystal. Wave properties of atoms result in zero vibrations in ordinary crystals, completely destroy crystalline lattice in quantum liquids, and allow atoms to move from one lattice site to another in quantum crystals, at the same time maintaining the individuality of crystal lattice sites.



Quantum motion of atoms from sites to sites can be revealed not only in vacancies-containing crystals, but also in crystals consisting of different sorts of atoms, and in particular in substitution-type alloys, in which the elementary act of motion is constituted by the tunneling exchange of places between two atoms of different sorts.

A theoretical analysis indicates that quantum crystals must have unique properties which will make an object for research in the future. We will cite only one experimental fact which demonstrates beyond any doubt that quantum crystals are real and not just hypothetical objects. This fact relates to the investigation of properties of solid helium, i. e. helium under pressure. X-ray structural analysis makes it possible to measure the spacings between the sites in the crystal lattice of a solid. Such measurements were carried out for solid helium. If one knows the dimensions of unit cell and weight of a helium atom, one can calculate its density, which can also be found directly by weighing. It was found that the results obtained by the two methods are different, i. e. a crystal is lighter than it is expected to be. This expected value is calculated for a classical crystal, one with no vacancies. A quantum crystal contains vacancies, in other words unoccupied lattice sites, and thus it is lighter.

Investigation of diffusion in solid helium, which we have mentioned above, confirmed the standpoint which states that solid helium in the vicinity of absolute zero is a quantum crystal\* (see Fig. 19).

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\* Quantum concepts concerning the motion of vacancies and impurity atoms are of recent origin (A. F. Andreyev, I. M. Lifshits, 1969). They led to the development of the theory of quantum crystals.

## *Undamped macroscopic motions*

The motion of electrons in atoms is often cited as an example of undamped motion. If, however, an atom is excited, i. e. if the electrons are forced to move "faster than they normally must", it will take the atom a very short time to emit a photon and return to the ground state.

The motion at a macroscopic level, such as the flow of a liquid, the rolling motion of a wheel, or any other, must be maintained, because otherwise it is bound to stop, with energy of the macroscopic motion transformed to heat, "squandered" on random motion of atoms and molecules. Even heavenly bodies are gradually slowed down by the friction on the interplanetary gas, by the formation of tide waves, etc. Of course, this process is very slow, and there should be no fear that the slowed-down moon will fall onto the Earth in the foreseeable future. There are, however, exceptions to this seemingly general rule stating that a macroscopic motion cannot survive without external support. These are superfluidity of helium and superconductivity of metals. The superfluidity, discovered by P.L. Kapitsa, consists in the fact that helium's viscosity drops to zero at  $T = 2.19^\circ\text{K}$ , so that it flows through narrow capillaries without being slowed down. Superfluidity stipulates a series of unusual properties of this unique liquid. It was given an exhaustive explanation by L.D. Landau in a series of papers in 1941-1944. The superconductivity, discovered by Kamerlingh Onnes in mercury in 1911, consists in the fact that electric resistance of a metal drops to zero, which gives the metal a number of other astonishing properties. The microscopic theory of superconductivity was developed by J. Bardeen, J. Cooper and J. Schrieffer in 1956.

The distinctive feature of the behaviour of atomic particles, taking part in the superfluid or superconducting motion, consists in their strict coherence. All particles behave as a coherent whole, still remaining a liquid capable of flowing. A special term was introduced to describe this state: the *condensate*. Formation of a condensate distinguishes superfluidity and superconductivity from other macroscopic motions, where the observed effects are produced by randomly moving particles or quasiparticles forming huge associations. The motion of a column of soldiers as compared to chaotic shuttling of pedestrians in the street can serve as a very rough model of the superconducting or superfluid motion. We want to emphasize that both superfluidity and superconductivity are *macroscopic* phenomena. They can be recorded directly by macroscopic instruments. At the same time, both these phenomena are *quantum* in nature. In fact, the possibility to observe quantum laws of motion in a macroscopic experiment is offered only by superconductivity and superfluidity. Nowadays penetration of quantum physics into the solid state physics and even into technology blunts our feeling of surprise when we face the extraordinary, almost unimaginable world of quantum laws. Indeed, it would seem that nobody should show surprise at the discrete character of atomic energy levels, the fact lying at the foundation of the spectral analysis. Nevertheless, even to-day one is deeply impressed by the fact of direct observation in the macroscopic experiment of the discreteness of quantum states. This experiment was actually conducted: as a result of quantization of the superconducting motion in a cylindrical superconductor (Fig. 21), the magnetic flux inside it may assume only discrete values.

If specific properties of metals in the superconducting state are ignored, the two phenomena (superconductivity

and superfluidity) can be discussed in identical terms. As Landau put it, *superconductivity is superfluidity of a charged liquid*. But what is the nature of superfluidity?

As we have emphasized in the section "Phonons in helium", all possible motions of helium atoms on the microscopic scale are represented by sound waves, or phonons. Let us analyze this statement. It is clear

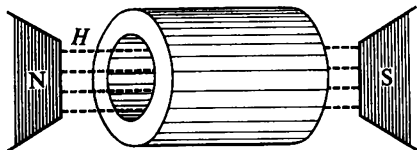


Fig. 21. The experiment that enables to detect quantization of a magnetic flux. Magnetic flux  $HS$ , passing through a superconducting hollow cylinder with the inner orifice area  $S$ , is equal to an integer number of flux quanta,  $\Phi_0 = 2\pi\hbar c/2e \approx 2.07 \cdot 10^{-7} \text{ gf} \cdot \text{cm}^2$ . The factor 2 in the denominator indicates that the superconducting current consists of Cooper electron pairs

that individual atomic motions are impossible due to interaction between the helium atoms. This interaction has both force and purely quantum nature (see above), and is naturally destroyed when temperature increases. A measure of this interaction can be found for helium as the temperature of disappearance of superfluidity, and for metals as the temperature of transition to the normal state. Therefore, interaction between particles creates a bound system ("a column of soldiers") which can move as a single entity. What then stands in the way of transformation of this ordered motion into the thermal motion, what precludes the "squandering" of the energy of ordered motion? Let us continue the analogy of the military column. The ordered motion of the column may be disrupted even if the

soldiers remain in the ranks. This will happen if the rows in the column start wavering and squiggling. The factor preventing it in the case of a marching column is the discipline. But what about the motion of helium atoms?

Assume for the sake of simplicity that the experiment is run at the absolute zero. Clearly, thermal motion appears when phonons are generated, because no other type of motion is possible in helium. And as any process in physics, the process of phonon generation is governed by the laws of momentum and energy conservation. An analysis demonstrates: a phonon can be generated if the velocity  $V$  of the flow exceeds the minimum velocity of a phonon, i. e.  $V > \min(\epsilon/p)$ . This condition was called the Landau criterion. In the case of helium  $\epsilon/p$  does not equal zero for any  $p$  (see Fig. 3 on p. 39), and therefore the motion with velocity below  $V_{\text{crit}} = \min(\epsilon/p)$  is not accompanied by dissipation of energy, proceeds with zero friction, and hence can last eternally without support. This analysis shows that although the superfluid motion cannot be reduced to that of quasiparticles, the possibility for it to exist is guaranteed by the properties of quasiparticles, phonons in this particular case, for which  $\min(\epsilon/p) \neq 0$ .

One must not forget, of course, that both the formation of the bound state of all helium atoms, and the inequality of  $\min(\epsilon/p)$  to zero, are of identical nature. If helium atoms were not bound, their motion energy would be  $p^2/2m$ , with the ensuing

$$\min \frac{p^2}{2mp} = 0.$$

A few additional remarks on superconductors must be added. An analysis that we have to omit shows that Bose-statistics is favourable to superfluidity. The

atoms of the heavier isotope of helium are bosons\*, while the electrons in a metal are fermions. This fact hindered understanding of the nature of superconductivity for quite a long time. Finally it was found that the transition to the superconducting state is provided by the formation of specific electron quasi-atoms, the so-called *pairs*, consisting of two electrons. The pairs are formed owing to slight attraction between the electrons, acting against the background of strong electrostatic repulsion and caused by exchange of phonons: one electron generates a phonon and the second one absorbs it. Note that, by atomic scale, a pair is fairly large. The correlated motion of electrons in a pair takes place over a distance which exceeds the atomic dimensions by a factor of, approximately, several thousand. This explains the surprising fact that weak attraction prevails over strong electrostatic repulsion of electrons. The pairs are bosons, and in forming the condensate, play the role of the building material of the superconducting flow.

The book began with the question: "What are the components of ...?" The superconducting flow is an ideal example of the intricate dialectical character of the concept of a structural unit. The pairs can only exist in an association of electrons. They are formed in a degenerated electron gas, each electron of a pair being a component of this gas. On the other hand, in a certain sense the motion of each pair is individualized, so that each pair carries the charge equal to twice the electron charge. This is unambiguously indicated by the value of the quantum of magnetic flux in the space enclosed in the superconductor. An analysis of quantization

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\* The lighter isotope of helium,  $\text{He}^3$ , whose atoms are fermions, was discovered, not long ago, to possess superfluidity. The nature of the superfluid transition in  $\text{He}^3$  was understood only very recently.

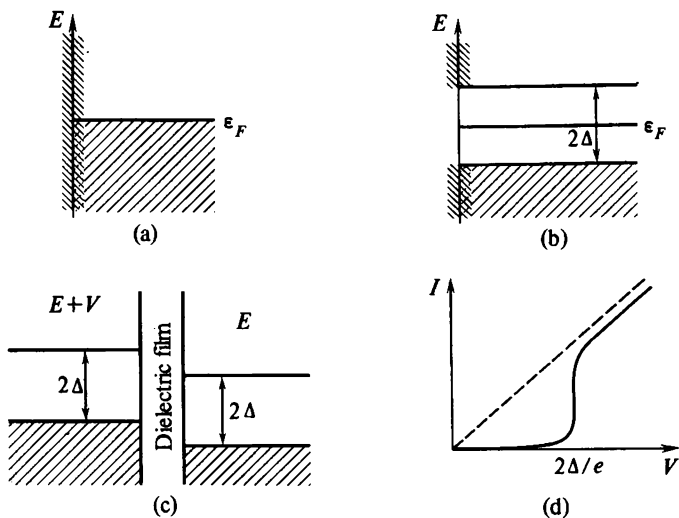


Fig. 22. Energy structure of a normal metal (a) and a superconductor (b). The energy gap above the Fermi energy, shown in the case of a superconductor, indicates the energy that must be expended in order to break a Cooper pair. Energy structure of a superconducting diode (c), i. e. of two identical superconductors separated by a thin ( $\approx 10^{-6}$  cm) dielectric film. The current-voltage characteristic of such a diode reveals the presence of the energy gap (d)

conditions demonstrates that one quantum of flux,  $\Phi_0$ , is given by

$$\Phi_0 = \frac{2\pi\hbar c}{Q},$$

where  $Q$  is the elementary charge of the carrier. The experiments mentioned above proved that  $Q = 2e$ , where  $e$  is the electron charge.

Finally, a few words concerning quasiparticles in

superconductors. Formation of pairs, i. e. states of electrons bound into pairs, modifies the energy spectrum of the metal. Excitation of the electron system in the superconducting state requires that a pair be "broken", therefore the spectrum contains an energy "gap", equal to the minimum amount of energy that can be accepted by the electron system of the superconductor. The presence of the "superconductor gap" in the electron energy spectrum is confirmed by a large number of experiments. The most straightforward of these experiments is the observation of the threshold value on the current-voltage characteristic of a superconducting tunnel diode (Fig. 22).

### *Concluding remarks. Solid state physics and molecular biology*

Speaking of solids, we mostly meant crystals, i. e. systems of particles regularly arranged in space. Not all of the described properties and features of crystals disappear when the system of particles is disordered. For instance, a metal retains its electric conduction even after it is melted, and its conductivity does not undergo any substantial change at the melting point. The factor essential for the appearance of free electrons is not the regular arrangement of atoms, but the separation of the valence electrons from nuclei due to the closeness of the atoms. The quantum language suitable for quantitative description of the properties of disordered systems is being developed at present, and considerable progress was achieved in recent years. In those cases, however, where the concept of quasiparticles cannot be applied, the whole system of views is so complicated that it does not appear possible so far to describe the obtained results without resorting to formulas.



Formation of simple lucid images, as a rule, signifies that a particular field of theoretical physics is brought to completion, and its ideas form a full self-consistent set. This is one of the characteristic features of *quantum* physics, in which lucidity of the images cannot serve as a guideline for developing new theories.

A crystal is an example of a highly organized matter. Given the structure of one unit cell of the crystal, with dimensions  $3 \cdot 10^{-8} \text{ cm} \times 3 \cdot 10^{-8} \text{ cm} \times 3 \cdot 10^{-10} \text{ cm}$ , we construct a macroscopic solid by the simple periodic extension of the cell along all three directions. A living tissue is also an example of high degree of organization. And if we outline a "hierarchy of values" for the organization of matter, we shall find that the organization of the living matter is much higher than that of the inanimate matter. Although the organization is coded in both cases in one molecule, however enormous it may be in the case of living matter on the scale typical of the inanimate one, the principle of construction of the former has nothing in common with the simple periodic extension.

Another point. In a crystal we easily single out the structure, i. e. the arrangement of atoms and molecules, and the motion which proceeds as if against the background of the structure. We have pointed out already that the arrangement of atoms prescribes the geography of the world of quasiparticles, which are carriers of motion. This separation is, of course, somewhat arbitrary, since the stability of a specific configuration is a function of not only static but also its dynamic properties. In this sense, the living matter appears to be organized quite differently. The stability of a living organism is maintained by unceasing motion, realized as metabolism of a living organism in the ambient medium.

We talk now about the motion which is life itself.

As life activities cease, any biological object can be described in standard physical terms. Such a system will relax, tending to the thermodynamical equilibrium in accord with the laws of physics. Description of motion in a system of this type is no different from that in any other object of the inanimate nature. In particular, one can determine the structure of a biological object, and consider the motion of atomic particles against the background of this structure. This standard language cannot describe the processes taking place in a *living* organism. To a certain extent, a biological object "functions" as a machine whose individual parts carry out quite definite functions ...

Yet, inspite of all the clearly visible features distinguishing crystals from the living matter, there is no doubt that the ideas and concepts generated in the course of investigating the quantum properties of solids will be successfully applied to the physics of life. And not only in the explanation of a number of mechanisms regulating transformation and transfer of energy in the living organisms. This is obvious. We also mean the solution of more fundamental problems, such as the problems of the origin of life, evolution of living matter, etc. This hope is based on our knowledge of a considerable number of qualitatively different approaches developed for the investigation of the properties of macroscopic systems, from a phenomenological description of a phenomenon which is rigorous enough but cannot pretend to elucidate its nature, to a detailed interpretation of the roles of all the macroscopic particles taking part in the phenomenon.

We have to warn the reader against too literal and therefore vulgarized interpretation of the above statements. Living organisms consist of elements (protein macromolecules, DNA molecules, etc.) each of which is an

extremely intriguing physical object, and physics of condensed state will undoubtedly play, and already plays, a very substantial role in understanding the properties and functions of such objects. Considerable success has already been achieved at this level. The transition from the molecular level to the subcellular, then cellular level and then to the organism as a whole reveals the specifics of the processes and phenomena taking place on the different "floors" of the biological structures. It seems that a physical description of these specific processes and phenomena requires that certain qualitatively new concepts be developed, taking into account the peculiar character of kinetic phenomena in the conditions of rigid control of the whole biological system. In all likelihood, the solid state physics will help the development of these concepts not so much by its specific results as by setting examples of formulating new concepts in cases of transition from one level of organization of the matter to a higher one.

